The 2nd National Conference on Soft Computing and Cognitive Science

Gonbad Kavous University-18 & 19 May 2023

Conference Papers (English)

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Motivational Factors for ELT enhancement and Machine Learning prediction

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ABSTRACT

Motivation plays an important role in education to make the students an active subject in learning and significantly effect on their own learning and their peers. Moreover, teacher as one of the important lope of the educational chain could motivate the learners to play an active role in learning. Thus, the educational experts attempt using motivational factors to encourage the learners for getting better result at the end of the course. Though, the researchers think of a predicting technique to help learners at the initiation of the semester by motivational questionnaire and utilizing Machine Learning (ML). Current researchers selected a cohort of 99 students who selected General English course through random sampling from both sexes from the university context to fill the questionnaire that was in a 5-point Likert type scale using motivational factors. Usually, Linear Regression or Logistic Regression are used for analyzing data according to the dependent variables. But, in the present study data was analyzed using Support Vector Machine (SVM) which is the algorithm that is used for predicting students' performance in English course. Thus, (SVM) is compared with logistic regression (LR). The accuracy of 73.33% for LR and 96.66% for SVM, indicated a much higher accuracy in SVM method.

Keywords
Motivational Factors, Learning, Machine learning, Logistic Regression, Support Vector Machine

1. INTRODUCTION

Over time, technology-based teaching has become a dynamic part of the learning process, an input and an output. Moreover, these practices have become key building blocks that advance the building blocks of learning systems, improve the foundations of curricula, and play an important role in increasing effectiveness and ingenuity.
However, machine learning (ML) was introduced to education more than 30 years ago as a revolutionary educational tool, advancing education in nearly every field, and language teaching professionals are using it to learn how technology can We are advancing language learning into the 21st century that has arrived. Top high in his arena. ML is an innovative tool for fighting cancer, climate change and even terrorism (Tomei, 2012).
Though, as this innovative ML-based, education has come to exist in the third millennium, this new teaching and learning environment affect how teachers and students communicate and think of the result during semester, and the researchers in the current study claim that it can be used to predict a student’s performance at the initiation of the semester before the learners reach the end.
One of the major pedagogical techniques in which language learning reaches its peak is motivation both intrinsically and extrinsically. Accordingly, the result of a study of 102 Shiraz University Science students indicated that Integratively-motivated students have shown to use more memory and cognitive strategies than instrumentally motivated students Hassanpur (1999).
Similarly, Sedaghat (2001) examined the effects of attitudes, motivation, and skill level on the use of listening methods in 109 Iranian EFL female students. As for the motivational element, social areas were considered important as solitary contrasting spaces. Therefore, integrative language learners used more familiar systems than instrumentally arranged learners.
Endeavoring to find the probable effect of motivation and attitudes, a study conducted an investigation on Yemeni students to learn English. The research focuses on three motivational structures, namely instrumental motivation, integrated motivation, and personal motivation. The results indicated that both instrumental and personal motives are

important motivations for learning English among them. Though, integration motivation has the lowest impact on students' learning (Al-Atemi & Shuib, 2009).

As the literature review indicates most of the previous studies conducted have dealt with the classic concepts of integrative and instrumental motivation and almost no study has ever been focused on the effects of ML on language learning or identifying some problems existing in this field in the 21st century that technology reaches the pick of its existence. Therefore, the researchers in this study were delved the feasibility of predicting students' Passing or Failing status with the questionnaire on the motivation factors. Since, the role of technology in higher education is to strengthen human thinking and expand the educational process, not to limit it to a set of steps for content delivery, control, and evaluation. The experts confirm that with emergence of ML around the world and entering almost all the branches of knowledge especially education, language pedagogy is no exception; so, human beings expecting the separation between individuals and ML systems are sluggishly starting to erode. With ML systems the supplementing and improving human competences are not only coming to exist, but the development of human capabilities has also started the new era.

2. METHOD

2.1 Participants

Using random sampling procedure, a total of 99 B.A. students in general English classes at the second semester of 2021 were chosen as participants of the study. The selected subjects were from both genders, and they were recruited from Payame Noor University (PNU) and Azad University (AU) in Khuzestan province. They were quite varied in terms of demographic factors like age and background.

2.2 Instrumentation

Questionnaire administration was used as the primary means of data collection in the current study. To obtain a clear picture of viewpoint of university students, the questionnaire that was designed by Dordinejad, (2015) on the motivational factors administered to 99 B.A. students. The employed questionnaire's content has been determined as intrinsic and extrinsic motivation. All questions were in a 5-point Likert type scale ranging from strongly disagree (with the value of 1) to strongly agree (with the value of 5).

2.3 Data Collection Procedure

The study was initiated by administering the questionnaires to students' participant. As mentioned earlier, the questionnaire created consisted of 37 Likert-type items, and participants were expected to complete the questionnaire in 45 minutes. Prior to the administration, all participants were informed of the research objectives and guaranteed the confidentiality and anonymity of their responses. Furthermore, because the students' grades were needed and due to the time of the exams, reaching respondents at our convenience was actually impossible, therefore, this phase of the study endured over a period of four weeks to reach the time of exam and their convenience time. It is noteworthy to state that among all participants 53 of them were male and 46 were female.

2.4 Questionnaire's Statistical information

In this study, the researchers used a questionnaire that was designed and prepared by Dordinejad 2015 to evaluate motivational factors in learning English. The survey reliability was calculated using Cronbach's alpha method and obtained an acceptable coefficient of 0.937 for the whole questionnaire. This questionnaire consists of 37 questions that are a combination of intrinsic and extrinsic motivational factors. In the present study, this questionnaire was used to assess students' view points on motivational factors.

3. Data Analysis

In this section, by employing classification algorithms, we predict the status of the student's score in General English language course in two classes of "passing and failing". In this regard, we use Support Vector Machine (SVM) and logistic regression (LR). In the first step, we obtain the model using SVM and LR. In the second step, with the help of training data, we explain the relationship model between independent variables that are (answer to questionnaire's items) and dependent variables that are (Passing or Failing). In the third step, the accuracy of this prediction will be inspected with the test data. Finally, after training data, we examine their accuracy with the assistance of test data.

3.1 Findings of the Logistic Regression Algorithm

According to most statisticians, Ronald Fisher was the first to propose a logistics model. In 1938, when the famous priest Malthus was studying the worrying population growth of the time, and because the growth rate was assumed to be exponential; Fischer offered him a growth rate that is also used in the exponential logistics model. Later in 1945, Pierre Francois Verhulst, who studied population growth, introduced the logistic function (logit curve). In some medical studies, the dependent variable is zero and one.
Logistic regression is a statistical regression model for binary dependent variables. This model can be considered as a generalized linear model that uses the logit function as a link function and its error follows a polynomial distribution. This model is:

$$\text{logit}(\pi_i) = Ln\frac{\pi_i}{1-\pi_i} = \alpha + \beta_1 x_{1,i} + \cdots + \beta_1 x_{k,i} \qquad i = 1,2,\ldots,n$$

$$\pi_i = \Pr(Y_i = 1) = \frac{e^{\alpha+\beta_1 x_{1,i}+\cdots+\beta_1 x_{k,i}}}{1 + e^{\alpha+\beta_1 x_{1,i}+\cdots+\beta_1 x_{k,i}}}$$

The purpose is to use the most economic model to make accurate predictions of output for individual types. To achieve this goal, a model, which includes all the predictor variables that are useful in predicting the response variable is produced.
For the logistic regression algorithm, using the Split Data filter, we divide the data into two parts; 70% of the data for training and 30% for testing. By running the model in Rapid Miner software and uses the performance operator to check the validity of the model. It can be seen that the following results are obtained. Table 1:

Table 1: Predictive accuracy with logistic regression algorithm

	Amount 1	Amount 2	Percentage of classes	Total accuracy
Prediction 1	4	8	33.33	
Prediction 2	0	18	100	73.33

As Table 1 shows, the prediction of "Passing" amongst test data, 18 cases were correct and 0 incorrect case was reported. It is also predicted 4 cases correctly and 8 cases incorrectly in "Failing". The correct prediction percentage for code 2 means "Passing" is 100 and for code 1 means "Failing" is 33.33. The total accuracy of the model is 73.33%. The information of the value of MSE and coefficient of determination (R^2) of this model is shown in Table 2:

Table 2: Evaluation criteria of logistic regression algorithm

Criterion	Amount
MSE	٠,٢٦٧
R^2	0.48

As can be seen, the MSE is 0.267 and R^2 is 0.48. These two criteria can be used in comparison with other models.

3.2 Finding of the Support Vector Machine

The support vector machine (SVM) method is one of the machine learning methods proposed by Vapnik and his colleague in 90s (Smola & Scolkopf, 1998). To minimize the model error in this method, the principles known as structural risk minimization have been used; while other methods (such as: artificial neural network) use the principles of experimental risk minimization (Dibike, et al, 2001; Cristianini, and Shaw-Taylor, 2000). Generally, the SVM is used in two or multi-group classification and regression issues. Like many machine learning methods, the process in this model consists of two stages of training and testing. When the training phase was over, the generalizability of the trained model is evaluated using experimental data. In fact, the SVM estimates the regression function using a set of linear functions. In this function, the deviation of the predicted values from the actual value by ε is allowed (loss function). Finally, the best answer given by structural risk mitigation principles to the risk measured by the loss function (Samui, 2008).
In this section, we use SVM to fit curves for data. Now, using the Split Data filter, we divide the data into two parts: 70% for training and 30% for testing. With implementing the model in Rapid Miner software and using the Performance operator, it checks the validity of the model.

Table 3: prediction Accuracy with SVM algorithm

	Amount 1	Amount 2	Class percentages	Total accuracy
Prediction 1	٤	1	80	96.67
Prediction 2	٠	25	١٠٠	

As table3 is presented; in 4 cases the students "Failing" status was correctly predicted and in 1 case incorrectly. Also, 25 cases correctly and 0 case incorrectly predicted the students "Passing" status. The correct prediction percentage for "Failing" among test data is 80% and for "Passing" is 100%. The overall accuracy of the model is 96.67%. Examining the SVM algorithm, we understand that the MSE and the R^2 of this model are as follows:

Table 4: Support Vector Machine Algorithm Evaluation Criteria

Criteria	Amount
MSE	0.036
R^2	0.869

As can be seen in Table 4, the MSE is 0.036 and the R^2 equal 0.869. These two criteria can be used in comparison with other models.

3.3 Comparing the results of the two algorithms

In this section, we compared the results of "Passing or Failing" forecasting using LR and SVM algorithms. The comparing algorithm information is displayed in Table 5:

Table 5: Comparison of Algorithms

Algorithm	**Total accuracy**	**MSE**	R^2
LR	٧٣,٣٣	٠,٢٦٧	0.48
SVM	96.67	0.036	0.869

Table5 indicates that the highest accuracy in predicting of “Passing or Failing” is 96.67 which is associated with the SVM algorithm. On the other hand, considering that the value of MSE for SVM algorithm is less than MSE for LR. so SVM algorithm is better than LR algorithm in predicting student “Passing or Failing” status in English language course.

4. Discussion and Conclusion

There is no doubt that motivation plays an important role in language learning. Numerous studies by numerous researchers have proven that motivation (intrinsic and extrinsic) is a factor influencing second and foreign language learning. Similarly, the large-scale studies presented (Bradford, 2007; Dörnyei, 1998; Engin, 2009) confirm that motivation is becoming a determinant of successful second or foreign language learning.

Therefore, in this study, we investigated potential motivations for using ML to predict the pass or fail status of students in General English courses. Researchers used standard questionnaires to assess students' motivation to learn English and analyzed the data to determine student pass or fail results.

In summary, the potential of ML to achieve better educational outcomes is validated by the available research to be a revolutionary technology and a kind of panacea from failure.

Finally, as shown in Table 7, the 96.67% accuracy of the SVM algorithm was achieved by the researchers in this study with ML, showing much higher accuracy than LR.

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Binary Farsi sign language and recognition system of this language

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ABSTRACT

For a long time, creating new ways to communicate, especially in sensitive situations, has been one of the topics of researchers' attention. In this article, we are going to build a new pointing method that is dedicated to the Persian alphabet and also the recognition system of this language using artificial intelligence. By using five fingers and binary states, various states have been produced. A simple calculation to obtain the number of states, each of these states is assigned to one of the letters of the alphabet. The construction of these 32 modes follows this simple formula. 5^2=32, the number 5 means five fingers, the number 2 indicates being binary, and finally, we will have 32 different states. The goal is to use a simple method for communication, of course, the goal of the detection system is to use simple methods with fewer calculations for the computer.

Keywords
Recognition of sign language, SLR, Farsi sign language, binary

1. INTRODUCTION

Before the 16th century, people treated the Karolals very unpleasantly. In the 17th century, a finger alphabet was invented, similar to the alphabet used by today's crurals. And then in the 18th century, a Frenchman named Charles de Luppe invented a kind of sign language. This language was a new way to use a set of traditional hand and arm movements in a way that expresses personal thought. Sign language has been around since deaf people have been around the world.

According to the World Health Organization (WHO), there are more than 360 million people with hearing impairment. This number has increased to 466 million by 2020, and it is estimated that by 2050, more than 900 million people will be hearing impaired. According to the World Federation of the Deaf, about 300 sign languages (SL) are used worldwide. SL is a bridge between deaf and normal people. It is defined as a method of interaction for the hearing impaired through a series of hand gestures, postures, gestures, and facial expressions or gestures that correspond to letters and words in our real life. To communicate with deaf people, an interpreter is needed to translate real-world words and sentences. Therefore, deaf people can understand us or vice versa. Unfortunately, the deaf do not have a written form and have a severe lack of electronic resources. [1]

The current society has isolated the deaf due to the lack of voice communication or simple communication with the deaf community. Every year, 1400 deaf people are born in the country, most of them are from hearing families. As the name "sign language" suggests, in sign language gestures, or more precisely, movements, hand gestures, eye movements, and standing direction are used to convey the meaning. If, for example, the movement of the mouth or the general expression of the face changes, the meaning of the produced word can also change, or in other words another word is produced. The same principle is true for the movements of fingers and hands. The most important factor in communication between people is the ability to communicate through different methods such as speech, writing and sign language. The inability of deaf and hard of hearing people to communicate intimately, as well as the lack of conventional education according to the needs of the society, has caused the exclusion of these people.[2]

The need for an organized and unified SL was first discussed at the World Congress of Symbols in 1951. The British Deaf Association (BDA) published a book called Gestuno[3]. Gestuno is an international SL for the deaf that contains a vocabulary list of about 1500 signs. The name "Gastonu" was chosen referring to the gesture and unity. This language arises in Western and Middle Eastern languages. Gestuno is considered a pidgin of SLs with limited vocabulary. It was established in different countries including America, Denmark, Italy, Russia and Great Britain in order to cover the international meetings of the deaf. However, Gestuno cannot be considered a language for several reasons. First, no child or normal person grows up using this universal language. Second, it does not have a single grammar (their book contains only a set of signs without a grammar). Thirdly, there are less

number of experts who are fluent or professional in practicing this language. Finally, it is not used on a daily basis in any country, and people are unlikely to replace their national SL with this international one [4].
Many approaches have been proposed that are a combination of dynamic scheduling or hidden Markov (Starnet et al., using vision-based approaches based on HMMs, presented Real Time ASL Recognition. This system uses 40 characters participated in two suggestion methods. The first suggestion method used a single camera mounted on a table and achieved 92% accuracy. The second method used a camera mounted on a hat worn by the user. was installed and achieved an accuracy of 98%. Their methods were performed with the ASL continuous dataset, for which only hand movement features were used [5]
SLR essentially depends on what the translation of each hand gesture and posture included in SL is, and continues from sign pointing to the stage of producing text for normal people to understand for deaf people. To recognize any feature, the feature extraction step is an important step in the recognition system. It plays the most important role in symptom detection. They should be unique, normal and pre-processed. Many algorithms have been proposed to solve sign recognition, from traditional machine learning (ML) algorithms to deep learning algorithms. On the other hand, few researchers have focused on SLID [6].
Artificial Intelligence is the wide branch of computer science. It deals with developing smart machines that can operate without human interaction. Artificial Intelligence can be used in video games, mobile phones, cars, surveillance, etc. The concept of Artificial Intelligence was introduced back in 1950. Alan Turing, a mathematician and computer scientist designed a machine named as “Turing Machine”. This machine can test whether the computers can make decisions or not. The test can check the ability of machines to respond like humans.[7,8,9]
It’s nice to realize that we humans managed to enable machines with our own natural skills: learning by example and perceiving the outside world. The only problem is that significantly more time and effort is needed to teach computers how to “see” like us. But if we think about the practical purpose this capability already brings to organizations and businesses, the effort pays off. Underlying image and video-recognition models are neural networks, which are loosely modeled on how the brain processes information. Whether it’s a digital photo or sequence of video images, neural nets look for patterns in the pixels and build an increasingly abstract representation of what they see. With enough examples, neural nets “learn” to recognize people, objects, and how they relate.
Top video-recognition models currently use three-dimensional convolutions to encode the passage of time in a sequence of images, which creates bigger, more computationally-intensive models. To reduce the calculations involved, Han and his colleagues designed an operation they call a temporal shift module which shifts the feature maps of a selected video frame to its neighboring frames. By mingling spatial representations of the past, present, and future, the model gets a sense of time passing without explicitly representing it.[10,11,12]
For a long time, the invention of different algorithms and methods for communication has been of interest, for example, Morse code or ASL in this article is supported through the fingers of all the letters of the alphabet, which are made by putting the letters of the words together. People who are unable to speak can easily convey their meaning by using specific hand gestures that are modeled on binary 0 and 1. This language supports 32 characters, which is more suitable for Persian letters. It is also much easier for the system in the detection phase. We can use it to encrypt characters. Globally, wherever there is a deaf person, the manual of the communication transfer form can be found, although the method proposed in this article, in addition to not needing a manual, is very simple to learn, and the language recognition software is also intelligent. It is written synthetically for security applications when it is not possible to communicate with other normal methods and the relevant software will recognize it.

2. METHODS

Recognizing the Persian binary sign language alphabet using computer vision helps deaf and dumb people. This paper proposes a method that converts a set of 32 binary combinations representing the "open" and "closed" positions of five fingers into binary states. With this method, static and dynamic images can be converted, and binary numbers They are converted into letters. 32 modes are made through 5 fingers in two open and closed modes. In fact, the entire Persian alphabet is supported. By placing letters together, words are made and communication is possible.
We have a simple calculation to obtain the number of states that we assign each of these states to one of the letters of the alphabet. The construction of these 32 states follows this simple formula.

(1) 5^2=32

The number 5 means five fingers, the number 2 indicates binary, and finally we will have 32 different states. The language recognition system is designed in such a way that we place our hand in front of the camera, then each of these 32 modes that are shown in front of the camera, this program will recognize it through a calculation based on artificial intelligence and will display the corresponding character in the output. The ability to understand the shape and movement of hands can be a critical component in improving the user experience in various technology domains and platforms. For example, it could be the basis for understanding sign language and controlling hand gestures, and it could also enable the overlay of digital content and information on top of the physical world in augmented reality. MediaPipe Hands is a high-accuracy hand and finger tracking solution that uses machine learning (ML) to infer 21 3D landmarks of a hand from just one frame. While current state-of-the-art approaches mainly rely on powerful desktop environments for inference, our method achieves real-time performance on mobile and even multi-hand. In this research, the MediaPipe libraries and which does not need a dataset, it recognizes images live through special algorithms of the library.

3. CONCLUSIONS

Among the most important advantages of this method is to establish Persian sign communication with a simple method. Among the advantages are the lack of high facilities and advanced technology, simple training, specific to the Persian language, the use of a simple method and without the use of data sets in The system recognizes this language. People who are unable to speak and hear can communicate through this Persian language, which is easy to learn. In addition, in sensitive situations and access to a camera that does not record sound, we can use 5 fingers in binary form, meaning the presence and absence of fingers is necessary to indicate each letter. In binary, 0 means the absence of electricity and 1 indicates the presence of electricity. Now with 5 fingers and two states of 0 and 1 finger, we can make 32 states. Actually, 2 times 5 is equal to 32. The meaning of 2 is existence and non-existence, and the meaning of 5 is the same as 5 fingers.

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Designing a mathematical model for the optimal diet of cardiac patients in a fuzzy environment

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ABSTRACT

Our purpose in this paper is to present a proper diet model for cardiovascular patients in the fuzzy environment. In previous studies many efforts were made regarding preparation of a diet model appropriate for different age groups. But in most studies the daily nutrient intake decisions were made based on crisp data. By prescribing a diet based on crisp data, some of realities are neglected. For the same reason, we dealt with human diet problem through fuzzy approach. Also due to the importance of nutrition for individuals, especially patients with cardiovascular diseases, we have provided nutrition model as multi-objective fuzzy linear programming problem in which minimization of saturated fats, salt, monosaccharide and amount of received calory are considered as our objectives. First we review literature of fuzzy mathematics and then we will discuss nutrition model of cardiovascular patients in the fuzzy environment. Results indicated uncertainty about amount of nutrients and their intake affects diet quality making it more realistic.

Keywords
mathematical model, fuzzy environment, cardiac patients

1. INTRODUCTION

Generally, a problem exists regarding the inaccurate values of nutrients in foods because the amounts of nutrients available in a certain food are normally known but there is always a question of their exact amounts. If presence of a certain nutrient is doubted, it will be assumed that its amount is negligible which makes the problem of inaccurate near-zero amounts appears again. When food is treated and packed precisely and stably (like variety of oils, sugar, biscuits, etc); the amounts of their nutrients are mainly expressed certainly and with precision. For other foods, amounts of nutrients per 100 gram of a certain food (e.g. apple, white bread, potato) might vary within an interval; extent of these variations might be rather large. Amount of carbohydrates in an apple is a function of apple type and level of maturation. Even besides checking maturation level and type of apple, other variations associated with soil and growth conditions are present. Accordingly, necessity of incorporating fuzzy concept is further appreciated for expressing the respective values.Some studies have been conducted concerning application of fuzzy logic in nutrition; please refer to research Wirsam et al(e.g., [1]). They are Demonstrated that a nutrient intake can be described in a differentiated way and can be evaluated by employing fuzzy decision making. In paper mamat et al (e.g., [2]), they investigated human diet problem by fuzzy price and considered other factors as crisp; they have just minimized the costs. in addition, the important factor in people’s nutrition is the amounts of macro and micro nutrients contained in any unit of consumed foods, which are undoubtedly uncertain values.for example, is it possible to determine the total amount of cholesterol in 100 g of an egg? Surely the answer is negative because factors such as chickens breed, quantity and quality of their food as well as quality of their growth environment have significant impact on volatility of the related number. In another example and according to studies and researched performed in the Agriculture Department of United States, there is 2 grams of sugar per 100 grams of lettuce. For several reasons these numbers cannot be considered as the basis of decision makings. If the lettuce implanted in area with plenty of water , then due to increased water amount in the unit volume of this plant, the density of sugar available in 100 grams will be less than density of sugar available in 100 grams of the same lettuce samples cultivated in an area with less water. Therefore, the number related to sugar in 100 grams of lettuce should be considered as fuzzy number.On the other hand, it is not possible to determine a definite boundary for people’s maximal and minimal daily usage of micronutrients and macronutrients because factors such as geographical region, gender, age, etc. because fluctuation and obscurity in the usage amount of nutrients. For example, when it is said the maximal permissible daily usage amount of sodium is 2300 mg, this value must be considered as fuzzy number.In summary the reason for dealing with nutrition problem from fuzzy standpoint in this paper is great significance of applying suitable diet for specific patients including cardiovascular disease, because the daily required amount of these people to fats, carbohydrates, proteins, etc are dependent on patient's physical state. For this reason, one is not able to properly acquire an optimal nutrition combination using the classical approach. according to what mentioned above, risk will exist even if we follow nutritional instructions prescribed based on classic method. For example, if a heart disease patient is allowed to eat 100 grams of low-fat yogurt per day, both the term "low-fat "and previously mentioned issues are sufficient to look at optimal nutrition of these patients as a fuzzy concept.

Fuzzy logic was first proposed by Zadeh in 1965. Fuzzy set theory has been applied to many disciplines such as control theory and management sciences, mathematical modeling and industrial applications. The concept of fuzzy linear programming (FLP) on general level was first proposed by Tanaka et al [3]. Fuzzy linear programming (FLP) is a very useful and practical model for many real world problems. Concept of decision analysis in fuzzy environment was first proposed by Bellman and Zadeh [4]. Zimmermann [5,6] first used the max-min operator of Bellman and Zadeh to solve FLP problems. Other researchers used this operator, too. (e.g. [7,8]). Max-min operator has been used in solving other types of fuzzy programming. (e.g., [9, 5]) afterwards, many authors considered various types of the FLP problems and proposed several approaches for solving these problems. For example B. Jana and T. Kumar Roy examine the transportation model in fuzzy environment. Coefficients of target, supply, demand and capacity transfer functions were expressed as fuzzy numbers in their model [10].

In this paper, we discussed about Optimizing diet problem for cardiovascular disease in fuzzy environment and it formulates the research problem as a linear multi-objective fuzzy programming problem (MOFLPP) with mixed constraints in which right hand side of constraints are fuzzy numbers.and presents a suitable solution for it.Using Bellman and Zadeh's fuzzy decision-making process, the MOFLPP is converted into an equivalent crisp LPP. Then it is solved by simplex method. Next we have also considered MOFLPP with coefficients of objective as well as constraint functions and right hand sides of constraints are Triangular Fuzzy Number (TFN). Converting it into an equivalent crisp non-linear programming problem, it is also solved by fuzzy decisive set method. More ever, we have supposed diet Problem in the form of a fuzzy linear programming with four objective functions. All coefficients were assumed as triangular fuzzy numbers in this model. In the respective nutrition model which is at the form of a multi-objective fuzzy linear program, we attained good results for presenting an optimum diet through minimizing final saturated fats, salt, monosaccharide and amount of received calery in per day. in fuzzy approach by considering the parameters in their real values (through taking into account all possible states) would result in enhancement of nutritional quality. This causes the decisions to be more realistic in this regard.

2. DEFINITIONS AND PRELIMINARIES

Definition 2.1. (Triangular Fuzzy Number (TFN))
Let F (R) be a set of all triangular fuzzy number in a real line R . A triangular fuzzy number $\tilde{A}$ $(\in F(R))$ represented with three points $\tilde{A} = (a_1, a_2, a_3)$ .This representation is interpreted as membership functions. (Fig -1)[10].

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \le x \le a_3 \\ 0 & elsewhere \end{cases}$$

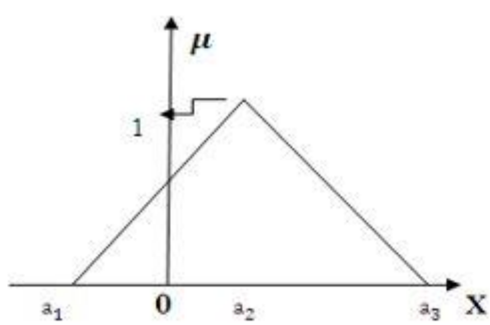

Fig-1 TFN $\tilde{A} = (a_1, a_2, a_3)$

It is parameterized by a triplet $\tilde{A} = (a_1, a_2, a_3)$ where a_1, a_3 are the lower and upper limits of support of $\tilde{A}$ and a_2 is the pick (or center) value of $\tilde{A}$ (Fig-1).
Remark 2.1 we consider $\tilde{0} = (0,\ 0,\ 0)$ as the zero triangular fuzzy number.
Definition 2.2 The left TFN $\tilde{A} = (a_1, a_2, a_2)$ is suitable to represent positive large or words with similar meaning. Provided that $a_2 > a_1$ it is represented by the following membership functions [10]:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & x \geq a_2 \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 0 & x \leq a_1 \end{cases}$$

Definition 2.3 The right TFN $\tilde{A} = (a_2, a_2, a_3)$ is suitable to represent positive small or words with similar meaning. Provided that $a_3 > a_2$. It is represented by the following membership functions[10]:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 & x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & x \geq a_3 \end{cases}$$

Remark 2.2 TFN $\tilde{A} = (a_1, a_2, a_3)$ is positive (negative) if $a_1 \geq 0$ $(a_3 < 0)$.

3. Multi-objective linear programming problems (MOLPP) with fuzzy Coefficients and Fuzzy Resources

The General Multi-Objective Linear Programming Problem (GMOLPP) with mixed constraints may be written as follows [10]:

$$\text{Minimize } Z = [Z^1, Z^2, Z^3, \dots, Z^K] \qquad (3.1)$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \quad i = 1,2,3, \dots, s_1$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad i = s_1+1, s_1+2, \dots, s_2$$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = s_2+1, s_2+2, \dots, s$$

$$x_j \geq 0 \;,\; j = 1,2,3, \dots, n$$

Where

$$Z^k = \sum_{j=1}^{n} c_j{}^k x_j \quad , k = 1,2,3, \dots, K$$

3.1 MOLPP with fuzzy Coefficients and Fuzzy Resources

In generality when the objective function's coefficients, technological coefficients and also right hand side of constraints are fuzzy numbers then (3.1) becomes [10]:

$$\text{Minimize } \tilde{Z} = [\tilde{Z}^1, \tilde{Z}^2, \tilde{Z}^3, \dots, \tilde{Z}^K] \qquad (3.2)$$

Subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \geq \tilde{b}_i \quad i = 1,2,3, \dots, s_1$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i \quad i = s_1+1, s_1+2, \dots, s_2$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j = \tilde{b}_i \quad i = s_2+1, s_2+2, \dots, s$$

$$x_j \geq 0 \;,\; j = 1,2,3, \dots, n$$

Where

$$\tilde{Z}^k = \sum_{j=1}^{n} \tilde{c}_j{}^k x_j \quad , k = 1,2,3, \dots, K$$

Assumption1: Fuzzy objective and constraints coefficients are considered as the following positive TFN's [10]:
Right TFN $\tilde{c}_j{}^k = (c_j{}^k, c_j{}^k, c_j{}^k + p_j{}^k)$ with tolerance $p_j{}^k > 0$ for objective function
$\sum_{j=1}^{n} \tilde{c}_j{}^k x_j$ For $k = 1,2,3, \dots, K$
Left TFN $\tilde{a}_{ij} = (a_{ij} - d_{ij}{}^0, a_{ij}, a_{ij})$ with tolerance $d_{ij}{}^0 < a_{ij}$ and $\tilde{b}_i = (b_i - b_i{}^0, b_i, b_i)$
With tolerance $b_i{}^0 < b_i$ for $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \geq \tilde{b}_i$ $\quad$ i $= 1,2,3, \dots, s_1$
Right TFN $\tilde{a}_{ij} = (a_{ij}, a_{ij}, a_{ij} + d_{ij}{}^0)$ with tolerance $d_{ij}{}^0 > 0$ and $\tilde{b}_i = (b_i, b_i, b_i + b_i{}^0)$
With tolerance $b_i{}^0 > 0$ for $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i$ $\quad$ i $= s_1 + 1, s_1 + 2, \dots s_2$
TFN $\tilde{a}_{ij} = (a_{ij} - d_{ij}{}^l, a_{ij}, a_{ij} + d_{ij}{}^r)$ with tolerance$d_{ij}{}^l < a_{ij}$, $d_{ij}{}^r > 0$
And $\tilde{b}_i = (b_i - b_i{}^l, b_i, b_i + b_i{}^r)$ With tolerance $b_i{}^l \langle b_i , b_i{}^r \rangle 0$
For$\sum_{j=1}^{n} \tilde{a}_{ij} x_j = \tilde{b}_i$ $\quad$ i $= s_2 + 1, s_2 + 2, \dots, s_r$. For the calculation of upper (U_k) and lower (L_k) bounds of the k-th $(k = 1, 2, 3 \dots K)$ objective function, we first construct the following eight sub-problems:

$$\text{Minimize } Z^{k1} = \sum_{j=1}^{n} c_j{}^k x_j \qquad (3.3)$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \qquad \text{i} = 1,2,3, \dots, s_1$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \text{i} = s_1 + 1, s_1 + 2, \dots, s_2$$
$$\sum_{j=1}^{n} a_{ij} x_j = b_i \qquad \text{i} = s_2 + 1, s_2 + 2, \dots, \text{s}$$
$$x_j \geq 0 \qquad \text{j} = 1,2,3, \dots, n$$

$$\text{Minimize} Z^{k2} = \sum_{j=1}^{n} (c_j{}^k + p_j{}^k) x_j \qquad (3.4)$$

Subject to same constraints of (3.3)

$$\text{Minimize } Z^{k3} = \sum_{j=1}^{n} c_j{}^k x_j \qquad (3.5)$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i - b_i{}^0 \qquad \text{i} = 1,2,3, \dots, s_1$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i + b_i{}^0 \qquad \text{i} = s_1 + 1, s_1 + 2, \dots, s_2$$
$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i - b_i{}^l \qquad \text{i} = s_2 + 1, s_2 + 2, \dots, \text{s}$$
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i + b_i{}^r \qquad \text{i} = s_2 + 1, s_2 + 2, \dots, \text{s}$$
$$x_j \geq 0 \qquad \text{j} = 1,2,3, \dots, n$$

$$\text{Minimize } Z^{k4} = \sum_{j=1}^{n} (c_j{}^k + p_j{}^k) x_j \qquad (3.6)$$

Subject to same constraints of (3.5)

$$\text{Minimize } Z^{k5} = \sum_{i=1}^{n} c_j{}^k x_j \qquad (3.7)$$

Subject to

$$\sum_{j=1}^{n} (a_{ij} - d_{ij}{}^{0}) x_j \ge b_i \qquad i = 1,2,3,\dots,s_1$$

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}{}^{0}) x_j \le b_i \qquad i = s_1+1, s_1+2,\dots,s_2$$

$$\sum_{j=1}^{n} (a_{ij} - d_{ij}{}^{l}) x_j \ge b_i \qquad i = s_2+1, s_2+2,\dots,s$$

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}{}^{r}) x_j \le b_i \qquad i = s_2+1, s_2+2,\dots,s$$

$$x_j \ge 0 \qquad j = 1,2,3,\dots,n$$

$$\text{Minimize } Z^{k6} = \sum_{i=1}^{n} (c_j{}^k + p_j{}^k) x_j \qquad (3.8)$$

Subject to same constraints of (3.7)

$$\text{Minimize } Z^{k7} = \sum_{i=1}^{n} c_j{}^k x_j \qquad (3.9)$$

Subject to

$$\sum_{j=1}^{n} (a_{ij} - d_{ij}{}^{0}) x_j \ge b_i - b_i{}^{0} \qquad i = 1,2,3,\dots,s_1$$

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}{}^{0}) x_j \le b_i + b_i{}^{0} \qquad i = s_1+1, s_1+2,\dots,s_2$$

$$\sum_{j=1}^{n} (a_{ij} - d_{ij}{}^{l}) x_j \ge b_i - b_i{}^{l} \qquad i = s_2+1, s_2+2,\dots,s$$

$$\sum_{j=1}^{n} (a_{ij} + d_{ij}{}^{r}) x_j \le b_i + b_i{}^{r} \qquad i = s_2+1, s_2+2,\dots,s$$

$$x_j \ge 0 \qquad j = 1,2,3,\dots,n$$

$$\text{Minimize } Z^{k8} = \sum_{i=1}^{n} (c_j{}^k + p_j{}^k) x_j \qquad (3.10)$$

Subject to same constraints of (3.9)

3.2 Fuzzy Programming Technique for the Solution of MOLPP with Fuzzy Coefficients and Fuzzy Resources. [10]

Let L_k and $U_k$be the lower and upper bound for the k-th objective, where L_k= aspired level of achievement for the k-th objective function, and U_k= highest acceptable level of achivement for the k-th objective function. When the aspiration levels for each objective have been specified, we formed a fuzzy model. Our next step is to transform the fuzzy model into a crisp model the Foregoing steps may be presented as follows:

Step-1: Solve the MOLPPs (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9) and (3.10) for each kth objectives($k = 1,2,3 \ldots, K$).

Step-2: From the results of step -1, determine the corresponding value for every objective function at each solution.

Step-3: Find upper and lower bounds (U_k i.e and L_k) for kth objective from the 8k objective values derived in step-2, as follows:

$$L_k = \min_{\substack{1 \le r \le K \\ 1 \le s \le 8}} Z^k(X^{(rs^*)}) \quad k = 1,2,3,\ldots,K \qquad (3.11)$$

$$U_k = \max_{\substack{1 \le r \le K \\ 1 \le s \le 8}} Z^k(X^{(rs^*)}) \quad k = 1,2,3,\ldots,K$$

Step-4: The initial fuzzy model becomes (in terms of aspiration levels with each objectives)

$$\text{Find } \{x_j \ge 0 : j = 1,2,3,\ldots,n\} \qquad (3.12)$$

$$So\ as\ to\ satisfy$$

$$Z_k \lessapprox L_k \qquad \mathrm{k} = 1,2,3,\ldots,K$$

$$\sum_{\mathrm{i}=1}^{n} a_{\mathrm{ij}} x_j \gtrapprox b_i \qquad \mathrm{i} = 1,2,3,\ldots,s_1$$

$$\sum_{i=1}^{n} a_{\mathrm{ij}} x_j \lessapprox b_i \qquad \mathrm{i} = s_1+1, s_1+2,\ldots,s_2$$

$$\sum_{i=1}^{n} a_{\mathrm{ij}} x_j \cong b_i \qquad \mathrm{i} = s_2+1, s_2+2,\ldots,s$$

Here the membership functions for the fuzzy constraints of (3.12) are defined as :(For K-th constraints$\tilde{G}_k$; $k = 1,2,3,\ldots,K$))

$$\mu_{\tilde{G}_k}(U_k) = \begin{cases} 0 & U_k < \sum_{j=1}^{n} c_j{}^k x_j \\ \dfrac{U_k - \sum_{j=1}^{n} c_j{}^k x_j}{\sum_{j=1}^{n} p_j{}^k x_j + N} & \sum_{j=1}^{n} c_j{}^k x_j \le U_k \le \sum_{j=1}^{n} (c_j{}^k + p_j{}^k) x_j + N_k \\ 1 & U_k \ge \sum_{j=1}^{n} (c_j{}^k + p_j{}^k) x_j + N_k \end{cases}$$

Where $N_k = U_k - L_k$, $\mathrm{k} = 1,2,3,\ldots,$K

(for the *ith* constraints $\tilde{C}_i$; $i = 1,2,3,\ldots,s_1$))

$$\mu_{\tilde{C}_i}(b_i) = \begin{cases} 0 & \sum_{j=1}^{n} a_{ij} x_j < b_i \\ \dfrac{\sum_{j=1}^{n} a_{ij} x_j - b_i}{\sum_{j=1}^{n} d_{ij}{}^0 x_j + b_i{}^0} & \sum_{j=1}^{n} (a_{ij} - d_{ij}{}^0) x_j - b_i{}^0 \le b_i \le \sum_{j=1}^{n} a_{ij} x_j \\ 1 & \sum_{i=1}^{n} (a_{ij} - d_{ij}{}^0) x_j - b_i{}^0 \ge b_i \end{cases}$$

(For the *ith* constraints $\tilde{C}_i\ ; i = s_1 + 1, s_1 + 2, \dots, s_2$))

$$\mu_{C_i}(b_i) = \begin{cases} 0 & \sum_{j=1}^{n} a_{ij}\,x_j \ > \ b_i \\ \dfrac{b_i - \sum_{j=1}^{n} a_{ij}\,x_j}{\sum_{j=1}^{n} d_{ij}{}^0\,x_j + b_i{}^0} & \sum_{j=1}^{n} a_{ij}\,x_j \ \le \ b_i \le \sum_{j=1}^{n} (a_{ij}+d_{ij}{}^0)x_j + b_i{}^0 \\ 1 & b_i \ge \sum_{j=1}^{n} (a_{ij}+d_{ij}{}^0)x_j + b_i{}^0 \end{cases}$$

(For t

$$\mu_{C_i}(b_i) = \begin{cases} \dots \sum_{j=1}^{n} d_{ij}{}^l\,x_j + b_i{}^r \\ \dfrac{\sum_{j=1}^{n} (a_{ij} + d_{ij}{}^r)x_j + b_i{}^l - b_i}{\sum_{j=1}^{n} d_{ij}{}^r\,x_j + b_i{}^l} \\ 0 \end{cases}$$

$$\sum_{j=1}^{n} (a_{ij} - d_{ij}{}^l)x_j - b_i{}^r \le b_i \le \sum_{j=1}^{n} a_{ij}\,x_j$$

$$\sum_{j=1}^{n} a_{ij}\,x_j \le b_i \le \sum_{j=1}^{n} (a_{ij} + d_{ij}{}^r)x_j + b_i{}^l$$

Step-5: Using the max-min operator (as Zimmermann [5]) crisp LPP for (3.2) is formulated as follow:

$$\text{Maximize} \quad \lambda \tag{3.13}$$

Subject to

$$\sum_{j=1}^{n} (c_j{}^k + \lambda p_j{}^k)x_j + \lambda(U_k - L_k) \le U_k \qquad k = 1,2,3,\dots,K$$

$$\sum_{j=1}^{n} (a_{ij} - \lambda d_{ij}{}^0)x_j - \lambda b_i{}^0 \ge b_i \qquad i = 1,2,3,\dots,s_1$$

$$\sum_{j=1}^{n} (a_{ij} + \lambda d_{ij}{}^0)x_j + \lambda b_i{}^0 \le b_i \qquad i = s_1 + 1, s_1 + 2,\dots,s_2$$

$$\sum_{j=1}^{n} (a_{ij} - (1-\lambda)d_{ij}{}^l)x_j + \lambda b_i{}^r \le b_i + b_i{}^r \qquad i = s_2 + 1, s_2 + 2,\dots,s$$

$$\sum_{j=1}^{n} (a_{ij} + (1-\lambda)d_{ij}{}^r)x_j - \lambda b_i{}^l \ge b_i - b_i{}^l \qquad i = s_2 + 1, s_2 + 2,\dots,s$$

$$0 \le \lambda \le 1\ ,\ x_j \ge 0 \quad j = 1,2,3,\dots,n$$

Remark 3.1: The constraints in problem (3.13) containing cross product terms λx_j, $(j = 1,2,3,\dots,n)$ Which are not convex! Therefore the solution of this problem requires the Special approach adopted for solving general non-convex application problems. It may be solved by fuzzy decisive set method. (Sakawa and Yano [11]). This method is based on the idea that, for a fixed value of λ , the problem (3.13) is linear programming problems. Obtaining the optimal solution λ^* to the problem (3.13) is equivalent to determining the maximum value of λ so that the feasible set is nonempty. The algorithm of this method for the problem (3.13) is presented below.

3.3 Algorithm

Step-1: Set λ =1 and test whether a feasible set satisfying the constraints of the Problem (3.13) exists or not, using phase one of the Simplex method(see [12]). If a feasible set exists, set λ=1, otherwise, set $\lambda^l = 0$ and λ^R=1 and go to the next step.

Step-2: For the value of $\lambda = \frac{\lambda^l + \lambda^R}{2}$ update the value of λ^l and λ^R using the bisection method as follows:
$\lambda^l = \lambda$ If feasible set is nonempty for λ and $\lambda^R = \lambda$, if feasible set is empty for λ.
Consequently, for each λ, test whether a feasible set of the problem (3.13) exists or not using has one of the Simplex methods and determine the maximum value of λ^* satisfying the constraints of the problem (3.13).

Numerical Example 1:

$$\begin{aligned} &\text{Minimize } \tilde{Z}^1 = \tilde{c}_1{}^1 x_1 + \tilde{c}_2{}^1 x_2 \qquad (1)\\ &\text{Minimize } \tilde{Z}^2 = \tilde{c}_1{}^2 x_1 + \tilde{c}_2{}^2 x_2 \end{aligned}$$

Subject to

$$\begin{aligned} \tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 &\le \tilde{b}_1\\ \tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 &\ge \tilde{b}_2\\ \tilde{a}_{31}x_1 + \tilde{a}_{32}x_2 &\le \tilde{b}_3\\ x_1, x_2 &\ge 0 \end{aligned}$$

Where $\tilde{c}_1{}^1 = \tilde{2} = (1.5,1.5,2)$; $\tilde{c}_2{}^1 = \tilde{1} = (.5,.5,1)$; $\tilde{c}_1{}^2 = \tilde{1} = (1,1,2)$, $\tilde{c}_2{}^2 = \tilde{3} = (3,3,3.5)$ Respectively for objective coefficients $\tilde{a}_{11} = \tilde{2} = (2,2,2.5)$; $\tilde{a}_{12} = \tilde{5} = (5,5,5)$; $\tilde{a}_{21} = \tilde{1} = (0,1,1)$; $\tilde{a}_{22} = \tilde{3} = (1.5,3,3)$; $\tilde{a}_{31} = \tilde{3} = (3,3,4)$, $\tilde{a}_{32} = \tilde{1} = (1,1,1)$ Respectively for technological coefficients, $\tilde{b}_1 = \widetilde{20} = (20,20,23)$, $\tilde{b}_2 = \tilde{3} = (2,3,3)$, $\tilde{b}_3 = \widetilde{30} = (30,30,35)$ respectively for constraint goals. To solve the problem (1), we first solve the following sixteen sub-problems:

Minimize $Z^{11} = 1.5x_1 + .5x_2$ (1.1)
Subject to

$$\begin{aligned} 2x_1 + 5x_2 &\le 20\\ x_1 + 3x_2 &\ge 3\\ 3x_1 + x_2 &\le 30\\ x_1, x_2 &\ge 0 \end{aligned}$$

Minimize $Z^{12} = 2x_1 + x_2$ (1.2)
Subject to same constraints of (1.1)

Minimize $Z^{13} = 1.5x_1 + .5x_2$ (1.3)
Subject to

$$\begin{aligned} 2x_1 + 5x_2 &\le 23\\ x_1 + 3x_2 &\ge 2\\ 3x_1 + x_2 &\le 35\\ x_1, x_2 &\ge 0 \end{aligned}$$

Minimize $Z^{14} = 2x_1 + x_2$ (1.4)
Subject to same constraints of (1.3)

Minimize $Z^{15} = 1.5x_1 + .5x_2$ (1.5)
Subject to

$$\begin{aligned} 2.5x_1 + 5x_2 &\le 20\\ 1.5x_2 &\ge 3\\ 4x_1 + x_2 &\le 30\\ x_1, x_2 &\ge 0 \end{aligned}$$

Minimize $Z^{16} = 2x_1 + x_2$ (1.6)
Subject to same constraints of (1.5)

Minimize $Z^{17} = 1.5x_1 + .5x_2$ (1.7)
Subject to

$$\begin{aligned} 2.5x_1 + 5x_2 &\le 23\\ 1.5x_2 &\ge 2\\ 4x_1 + x_2 &\le 35\\ x_1, x_2 &\ge 0 \end{aligned}$$

Minimize $Z^{18} = 2x_1 + x_2$ (1.8)
Minimize $Z^{22} = 2x_1 + 3.5x_2$

Minimize $Z^{21} = x_1 + 3x_2$ (1.9)
Subject to

$$\begin{aligned} 2x_1 + 5x_2 &\le 20\\ x_1 + 3x_2 &\ge 3\\ 3x_1 + x_2 &\le 30\\ x_1, x_2 &\ge 0 \end{aligned}$$

Minimize $Z^{22} = 2x_1 + 3.5x_2$ (1.10)
Subject to same constraints of (1.9)

Minimize $Z^{23} = x_1 + 3x_2$ (1.11)
Minimize $Z^{24} = 2x_1 + 3.5x_2$

Subject to

$$2x_1 + 5x_2 \le 23$$
$$x_1 + 3x_2 \ge 2$$
$$3x_1 + x_2 \le 35$$
$$x_1, x_2 \ge 0$$

Subject to same constraints of (1.11)

Minimize $\quad Z^{25} = x_1 + 3x_2 \quad$ (1.13)

Subject to

$$2.5x_1 + 5x_2 \le 20$$
$$1.5x_2 \ge 3$$
$$4x_1 + x_2 \le 30$$
$$x_1, x_2 \ge 0$$

Minimize $\quad Z^{26} = 2x_1 + 3.5x_2 \quad$ (1.14)

Subject to same constraints of (1.13)

Minimize $\quad Z^{27} = x_1 + 3x_2 \quad$ (1.15)

Subject to

$$2.5x_1 + 5x_2 \le 23$$
$$1.5x_2 \ge 2$$
$$4x_1 + x_2 \le 35$$
$$x_1, x_2 \ge 0$$

Minimize $\quad Z^{28} = 2x_1 + 3.5x_2 \quad$ (1.16)

Subject to same constraints of (1.15)

Solutions of the sub-problems ((1.1) – (1.16)) are shown in Table 1:

Table 1: the optimal solutions of the sub-problems ((1.1) – (1.16))

$X^{11*} = (0.2)$	$X^{21*} = (0.2)$
$X^{12*} = (0.2)$	$X^{22*} = (0.2)$
$X^{13*} = (0,1.3333)$	$X^{23*} = (,1235.,6255)$
$X^{14*}=(0,1.3333)$	$X^{24*} = (0.,6667)$
$X^{15*} = (,1484.,9505)$	$X^{25*} = (0.2)$
$X^{16*} = (0.1)$	$X^{26*} = (0.2)$
$X^{17*} = (,1235.,6255)$	$X^{27*} = (0.1,3333)$
$X^{18*} = (0.,6667)$	$X^{28*} = (0.1,3333)$

So

$$L_1 = \min_{\substack{1 \le r \le 2 \\ 1 \le s \le 8}} \{Z^1(X^{rs*})\} = 0.33335 \qquad U_1 = \max_{\substack{1 \le r \le 2 \\ 1 \le s \le 8}} \{Z^1(X^{rs*})\} = 1$$

$$L_2 = \min_{\substack{1 \le r \le 2 \\ 1 \le s \le 8}} \{Z^2(X^{rs*})\} = 2 \qquad U_2 = \max_{\substack{1 \le r \le 2 \\ 1 \le s \le 8}} \{Z^2(X^{rs*})\} = 6$$

According to step-4 formulating membership functions and following step-5, Crisp LPP of (1) is formulated as follows:

Max $\quad \lambda$

Subject to

$$(1.5 + .5\lambda)x_1 + (.5 + .5\lambda)x_2 + .66665\lambda \le 1 \qquad (1.17)$$
$$(1 + \lambda)x_1 + (3 + 0.5\lambda)x_2 + 4\lambda \le 6$$
$$(2 + .5\lambda)x_1 + 5x_2 + 3\lambda \le 20$$
$$(1 - \lambda)x_1 + (3 - 1.5\lambda)x_2 - \lambda \ge 3$$
$$(3 + \lambda)x_1 + x_2 + 5\lambda \le 30$$
$$0 \le \lambda \le 1, \quad x_1, x_2 \ge 0$$

The problem (1.17) may be solved by the fuzzy decisive set method. For λ=1, the problem can be written as:

$$2x_1 + x_2 \le 0.33335 \quad (1.18)$$
$$2x_1 + 3.5x_2 \le 2$$
$$2.5x_1 + 5x_2 \le 17$$
$$1.5x_2 \ge 4$$
$$4x_1 + x_2 \le 25$$
$$x_1, x_2 \ge 0$$

Since the feasible set is empty, by taking $\lambda^l = 0$, $\lambda^R = 1$, the new value of ($\lambda = \frac{0+1}{2} = 0.5$) is tired. For $\lambda = 0.5$ the problem (1.17) can be written as:

$$1.75x_1 + .75x_2 \le 0.666675 \quad (1.19)$$
$$1.5x_1 + 3.25x_2 \le 4$$
$$2.25x_1 + 5x_2 \le 18.5$$
$$0.5x_1 + 2.25x_2 \ge 3.5$$
$$3.5x_1 + x_2 \le 27.5$$
$$x_1, x_2 \ge 0$$

Since the feasible set is empty, by taking $\lambda^l = 0$, $\lambda^R = 0.5$, the new value of $\lambda = \frac{0+.5}{2} = 0.25$, is tired and so on. The following values of λ are obtained in the next 23^{th} iterations:

λ = 0.25; λ = 0.375; λ = 0.3125; λ = 0.28125; λ = 0.265625; λ = 0.2734375;
λ =0.2773438; λ =0.2792969; λ =0.2802734; λ =0.2797852; λ =0.2800293;λ =0.2799072;
λ = 0.2798462; λ =0.2798767; λ =0.2798615; λ =0.2798538; λ = 0.27985

Consequently, we obtain the optimal value $\lambda^* = 0.27985$ at the 19^{th} iteration by using the fuzzy decisive set method and solutions of the problem (1) are $x_1^* = 0$, $x_2^* = 1.2711458$, $Z^{1*} = 0.6355729$, $Z^{2*} = 3.8134374$ and aspiration Level $\lambda^* = 0.27985$.

4. APPLICATION IN DIET PROBLEM FOR CARDIOVASCULAR DISEASE:

According to most experts of nutrition science, the main cause of heart disease as well as hypertension is inappropriate nutrition. Using Fast-foods as well as lack of exercise has caused cardiovascular disease in early ages [13]. But with little change in the nutrition plan and of course following the nutrition plan we can decrease risk of these common diseases in our family forever. According to report of United States Heart Association, having a balanced diet by minimizing final saturated fats, salt, monosaccharide and amount of received calery in per day is recommended to prevent these diseases [14].There are some limitations of using nutritious substances for cardiovascular patients. As we mentioned in section 1, in short the optimal nutrition pattern of heart disease patients are modeled as multi-objective fuzzy linear programming problem .We also consider minimum and maximum amount of using minerals and vitamins as some limitations. A general Multi-Objective nutrition model with mixed constraints, written as follows:

$$\text{Minimize } \tilde{Z}^1 = \sum_{i=1}^{n} \tilde{c}_j^{\,1} x_j \quad (4.1)$$
$$\text{Minimize } \tilde{Z}^2 = \sum_{i=1}^{n} \tilde{c}_j^{\,2} x_j$$
$$\text{Minimize } \tilde{Z}^3 = \sum_{i=1}^{n} \tilde{c}_j^{\,3} x_j$$
$$\text{Minimize } \tilde{Z}^4 = \sum_{i=1}^{n} \tilde{c}_j^{\,4} x_j$$

Subject to

$$\sum_{i=1}^{n} \tilde{a}_{ij}x_j \geq \tilde{b}_i \qquad i = 1,2,3,\dots,m$$

$$\sum_{i=1}^{n} \tilde{a}_{ij}x_j \leq \tilde{d}_i \qquad i = 1,2,3,\dots,m$$

$$x_j \geq 0,\ j = 1,2,\dots,n$$

Where:
x_j*: 100 g of food j eaten per day.*
$\tilde{c}_j^{\,1}$**:** *Approximate amount of calorie in 100 g of food j.*
$\tilde{c}_j^{\,2}$*: Approximate amount of saturated fats in 100 g of food j.*
$\tilde{c}_j^{\,3}$**:** *Approximate amount of Sugar in 100 g of food j.*
$\tilde{c}_j^{\,4}$**:** *Approximate amount of total sodium in 100 g of food j.*
$\tilde{a}_{ij}$**:** *The amount of nutrient i in 100 g of food j.*
$\tilde{b}_i$*: The required daily amount of nutrient i.*
$\tilde{d}_i$*: The maximum daily amount of nutrient i.*
M: The number of nutrients.
N: The number of foods.

For patients with normal weight, it is recommended to receive about 28 kcal of energy for every kilogram of patient weight [15]. Every gram of protein per gram of carbohydrate produce 4 kcal of energy. Also per gram of fat can produce about 9 kcal of energy [15]. According to studies, about 50% of intake energy should be provided by carbohydrate (Tolerable upper intake level 65% of intake energy), 5% from monosaccharide, 15% from protein and about 30% from fats. Because saturated oils are harmful to human health , so less than 10% of saturated fats should be used and the rest should be of unsaturated fats such as omega3 and omega 6[15]. In heart disease patients the sodium consumption is less than 2000mg. Also daily consumption of 20 to 35 grams of fiber is recommended. The problem (4.1) is a multi-objective fuzzy linear programming problem (MOFLPP). It can be solved as before.

Numerical Example 2:

In the following example 20 useful food are considered for the heart disease patients including a variety of fruits, low fat dairy products, meat, protein and useful vegetables. This foods are: Apple, Bread, Chicken, Fish, Garlic, Honey, Lettuce, Low-fat cheese, Low-fat yogurt, Low-fat milk, Olive oil, Orange, Pomegranate, Potato, Soybeans, Spinach, Tangerines, Tomatoes, Walnut, White-rice. The maximal and minimal values of required daily amounts of nutrients are included in table 2 for poorly active women with body mass index of 25 kg/m^2. Though the values in table 2 are presented by crisp numbers but they have been applied as triangular fuzzy numbers in the fuzzy linear programming problem. for 1900 Kcal of energy, at 4Kcal/g, 65%of calories correspond to maximum of Approximately 308 g of carbohydrate, 15%of calories correspond to maximum of Approximately 71 g of protein .while for 1900 Kcal of energy, at 9Kcal/g ,30% of calories correspond to a maximum of Approximately 63 g of fat(21.11 g of saturated fats and 52.77 g of unsaturated fats).table 2 shows the minimum ,maximum and actual nutrient requirements. The limitation related to amount of nutrient intake which is undefined in table 2 will be disregarded.

Table 2: approximate amount of nutrient and calory requirement per day(as [16])
(poorly active Female, 55 years old, sedentary, BMI 25 kg/m2)

Nutrients	Min	Max	Nutrient	Min	Max
Energy(kcal/d)	*1900*	*N.D*[1]	*Vitamin E(mg/d)*	*12*	*400*
Carbohydrat(g/d)	*130*	*322*	*Thiamin(B1)(mg/d)*	*1*	*N.D*
Total fiber(g/d)	*20*	*35*	*Folate(B9)(µg/d)*	*400*	*1000*
Sugar(g/d)	*47*	*124*	*Vitamin B12(µg/d)*	*2.4*	*N.D*
Saturated Fat(g/d)	*N.D*	*21.11*	*Calcium(mg/d)*	*1200*	*2500*
unSaturated Fat(g/d)	*42.2*	*52.77*	*Iron(mg/d)*	*18*	*45*
Total Omega-3(mg/d)	*1300*	*N.D*	*Mangnesium(mg/d)*	*320*	*350*

1:N.D= "Amount not determinable due to lack of data of adverse effects. Source of intake should be from food only to prevent high levels of intake.

Protein(g/d)	*46*	*71*	*Selenium(µg/d)*	*55*	*400*
Vitamin A(IU)	*2333*	*10000*	*Sodium(mg/d)*	*1500*	*2000*
Vitamin C(mg/d)	*75*	*2000*			

As shown in Table 2, approximate amount of nutrients and Energy requirement per day is given by crisp numbers. Decision variables, the same amount of food consumed daily (in terms of 100 grams) and minimal and maximal permissible daily amounts for using vitamins and minerals are included as the constraints. In order to create Triangular fuzzy number by using crisp data, one needs sampling and evaluating fluctuation level which in turn requires research and laboratory work, not performed by any researcher in the literature as of today. Nonetheless, we are forced to accept the related raw data gathered by nutrition authorities and alimental industries with slight deviation due to variations in price, taste and amount nutrients in foods as well as lack of certain boundaries about maximal and minimal amount of daily intake of nutrients varieties for individuals. Therefore, if deviation of raw data "a" is assumed to be "d", the related fuzzy number will be as follows: $\tilde{a} = (a - d, a, a + d)$.

$$\text{Minimize } \tilde{Z}^1 = \sum_{j=1}^{20} \tilde{c}_j^{\,1} x_j \qquad (4.2)$$

$$\text{Minimize } \tilde{Z}^2 = \sum_{j=1}^{20} \tilde{c}_j^{\,2} x_j$$

$$\text{Minimize } \tilde{Z}^3 = \sum_{j=1}^{20} \tilde{c}_j^{\,3} x_j$$

$$\text{Minimize } \tilde{Z}^4 = \sum_{j=1}^{20} \tilde{c}_j^{\,4} x_j$$

Subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \geq \tilde{b}_i \qquad i = 1, 2, 3, \dots, 18$$

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{d}_i \qquad i = 1, 2, 3, \dots, 15$$

$$x_j \geq 0, \; j = 1, 2, \dots, 20$$

Where all Coefficients in this MOFLPP are given as Appendix.by using the previous method the optimal solution of the above Fuzzy Nutrition problem shown in table 3.

Table 3: The amount of optimal solution parameters of Fuzzy

x_1 =**0.53016**	x_7 =8.9517	x_{13} =0.72478	x_{19} =0.01388
x_2 =**1.87762**	x_8 =0.01435	x_{14} =1.91395	x_{20} =0.0421
x_3 =**0.01455**	x_9 =0.05149	x_{15} =0.10742	and aspiration Level
x_4 =**0.38211**	x_{10} =5.10327	x_{16} =0.07305	is:
x_5 =**1.59929**	x_{11} =0.46772	x_{17} =0.23004	$\lambda = 0.0003335$
x_6 =**0.13759**	x_{12} =0.41563	x_{18} =3.94401	

Since x_j is a 100 g of food j eaten per day, we have:

Table 4: the amount of foods(In terms of gram) in crisp and fuzzy solution

Food	*Fuzzzy solution*	*Crisp solution*
Apple	*53.016*	*51.999*
Bread	*187.762*	*181.79*
Chicken	*1.455*	*-*
Fish	*38.211*	*75.991*
Garlic	*159.929*	*46.993*

Honey	*13.759*	-
Lettuce	*895.17*	*1184.359*
Low-fat cheese	*1.435*	-
Low-fat yogurt	*5.149*	*285.997*
Low-fat milk	*510.327*	-
Olive oil	*46.772*	*44.509*
Orange	*41.563*	-
Pomegranate	*72.478*	-
Potato	*191.395*	*306.574*
Soybeans	*10.742*	-
Spinach	*7.305*	-
Tangerines	*23.004*	-
Tomatoes	*394.401*	*459.205*
Walnut	*1.388*	-
White-rice	*42.1*	*154.484*

Table 5: the optimal solution for the Calery, Saturated Fat,sugar, Sodium

	Calory(Kcal)	*Saturated Fat(g/day)*	*Sugar(g/day)*	*Sodium(g/day)*
Crisp	*1900.0012*	*10.35322*	*73.17447*	*1.68043*
Fuzzy	*1974.1067*	*11.91002*	*100.520124*	*1.5001*
Difference	*74.10555*	*1.5568*	*27.345654*	*0.18033*

Table 4 shows the amount of foods in crisp and fuzzy solution.the main objectives of research that are to optimize the Calery,Saturated Fat,Sugar and Sodium (low- Calery,low-Saturated Fat,low-Sugar and low-Sodium) have been achieved.while table 6 Shows the intake of the nutrients by using the optimal cardiovascular diet in crisp and fuzzy solution.table 5 shows value of the objective functions.comparison between crisp and fuzzy solution,the results showd significant solution.the difference solution between normal diet problem using the model of minimize Calery,Saturated Fat,Sugar and Sodium diet problem and diet problem in fuzzy environment using the model of MOLPP with fuzzy objective cofficient or listed as the following:

1) There are 10 kinds of food in crisp solution and 20 kinds of food in the fuzzy solution.
2) The difference of the Calery is about 74 g .
3) Implementation of cardiovascular diet problem in fuzzy environment is not suggestive of total violation of its classic state; however, fuzzy approach –by considering the parameters in their real values (through taking into account all possible states) - would result in enhancement of nutritional quality. This causes the decisions to be more realistic in this regard.

Table 5: in take the nutrients by using the optimal human diet

Nutrients	*in Crisp solution*	*in Fuzzzy solution*
Energy(kcal)	*1900*	*1974.1067*
Carbohydrat(g)	*288.1344*	*302.0274*
Total fiber(g)	*34.61*	*34.1013*
Sugar(g)	*73.1745*	*100.5201*
Saturated Fat(g)	*10.3532*	*11.9100*
unSaturated Fat(g)	*47.9135*	*52.2174*
Total Omega-3(mg)	*1484.7318*	*1470.8741*
Protein(g)	*70.9999*	*70.4944*
Vitamin A(IU)	*10000*	*9793.2812*
Vitamin C(mg)	*142.4577*	*184.982*

Vitamin E(mg)	*12*	*12.0901*
Thiamin(B1)(mg)	*1.3095*	*1.6215*
Folate(B9)(µg)	*741.2661*	*681.3971*
Vitamin B12(µg)	*2.3999*	*2.4449*
Calcium (mg)	*1200*	*1492.1592*
Iron(mg)	*17.9999*	*18.1556*
Mangnesium(mg)	*349.9998*	*345.8773*
Selenium(µg)	*86.359*	*90.2593*
Sodium(mg)	*1680.4279*	*1500.0102*

macronutrients: the energy generated from this diet (in fuzzy environment) is about 1974 kcal .Taking into account table 5, amount of carbohydrate resulting from applying this diet is about 302g. In this nutrition diet, amount of sugar intake is approximately 100g,total fiber about 34g, Saturated Fat about 12g, unSaturated Fat about 52g, protein about 70g. Since for cardiovascular disease eating large amounts and complex carbohydrates instead of their simpler compounds is advised, we have attained this goal to some extent. Amount of Total fatty acids Omega-3 intake in this diet is around 1480mg which equals the needed daily value. About 5.5% of total received energy in this diet is provided by saturated fats.
Vitamins: The interesting point pertaining amount of receiving vitamin varieties in this diet is high value of vitamin A intake. It is around 9793 IU which might not be a great value. The amount of vitamin C intake is roughly 185mg that is a good value. By applying this diet, nearly 681µg is received which is significant for preventing certain cancers.
Minerals: According to table 5, one realizes that amount of receiving calcium about 1492mg that is a good value. amount of iron intake is amount 18mg and amount of Mangnesium about 346mg intake is amount that equals approximately maximum allowed daily intake of this mineral. By applying this diet is also received 90 µg of Selenium which is an ideal value due to essential role of this mineral in curing and preventing many diseases. Around 1500mg of Sodium is present in this diet that is a suitable value for controlling hypertension and preventing carbohydrates diseases.

5. Conclusion

Implementation of carbohydrates diseases diet problem in fuzzy environment is not suggestive of total violation of its classic state; however, fuzzy approach by considering the parameters in their real values (through taking into account all possible states) would result in enhancement of nutritional quality. This causes the decisions to be more realistic in this regard. Comparison of the results of the previous models and the proposed model proposed using the multi-objectives fuzzy programming problem in this study, the effectiveness of the proposed model is clearly evident. As observed before, numbers used in the optimal nutrition model of carbohydrates diseases diet problem in the fuzzy environment as fuzzy numbers based on which the subject of uncertainty in the previous dada were covered and considered with a real aspect of nutrition issue. In some cases there are misconceptions about the comparison of "fuzzy logic" and "reality". For example, is solving of various problems closer to reality in fuzzy environment or in classical environment? Are answers obtained in the fuzzy environment close to the answers obtained in the classical environment? In this research and similar studies we found that real world phenomena should be considered in the fuzzy environment due to their uncertainty. Also when earlier aspects are changed, then the decision making and action criteria should be considered in the fuzzy environment and the second question should be raised in another way. Are obtained answers in the classic environment close to the real answers?

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T-130

Consciousness and attention: an ontological study

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ABSTRACT

The aim of this study was to achieve a perspective about the nature of consciousness and attention, and also the relationship between these two. In order to achieve this aim, this paper is written as an ontological study by analyzing theories and concepts of consciousness and attention. Analyzes and results show us that in order to study the nature of consciousness, we must consider attention, which brings us to conscious attention. Conscious attention is involved in many of our cognitive abilities, such as memorizing, learning, planning, decision-making, focusing and self-awareness.

Keywords
Consciousness, Attention, Cognition, Ontology, Conscious attention

1. INTRODUCTION

Although mind and body are somehow united in the human person, still are ontologically entirely distinct. [1]. so this study isn't about the body and its relationship with the mind, it actually is an attempt to achieve a perspective about the nature of consciousness and attention. Awareness is an outcome (a conscious state) which is linked to the mechanism of attention, in many theories. [2]. so in order to study the nature of consciousness, attention should be studied first.

Historically, the concept of "attention" was the focus of philosophers and psychologists in the late 19th century. However, after that, it was less studied and discussed specially among psychologists, because the behaviorists regarded all internal processes. Attention returned to its place again following the publication of Broadbent's book – Perception and Communication – in 1958, and it has remained the focus of cognitive and philosophical studies ever since. [3].

2. ATTENTION

As Pashler pointed out: "Attention has long posed a major challenge for psychologists." [4]. William James says about attention: "Everyone knows what attention is. It is the taking possession of mind, in clear and vivid form, of one out of what seem several simultaneously possible objects or trains of thoughts. ... It implies withdrawal from somethings in order to deal effectively with others." [5].

Although many definitions have been presented for attention, there isn't such a specific definition for it. And unfortunately, psychologists still find attention difficult to define. The problem is that attention is not a single concept, but an umbrella term for a variety psychological phenomena. [6].

2. 1. WHAT IS ATTENTION?

Attention is most commonly used to refer to selectivity of processing. [3]. Attention is a mechanism for the selection of information. Awareness in many theories is linked to this mechanism. [2]. As a result of attention performance, a limited amount of information from sensations, memories and other cognitive processes will be selected and processed. [7]. Most psychologists agree that the brain has a limited ability to process information at once, and as a result, the brain needs a selective mechanism to function effectively, so that it can focus on specific information among the masses of received information. [8]. The main part in most definitions of attention, given in different references is that attention is a selective mechanism or means to be able to react better to the received (input) information.

2. 2. HOW DOES ATTENTION WORK?

Attention includes not only conscious, but also unconscious processes. [9]. Attention allows us to focus on the stimuli that interest us and focus less on the stimuli that doesn't interest us. Heightened attention also paves the way for memory processes. It means that the selected information which we paid more attention to, will be remembered so much easier than the information we ignored. [10].

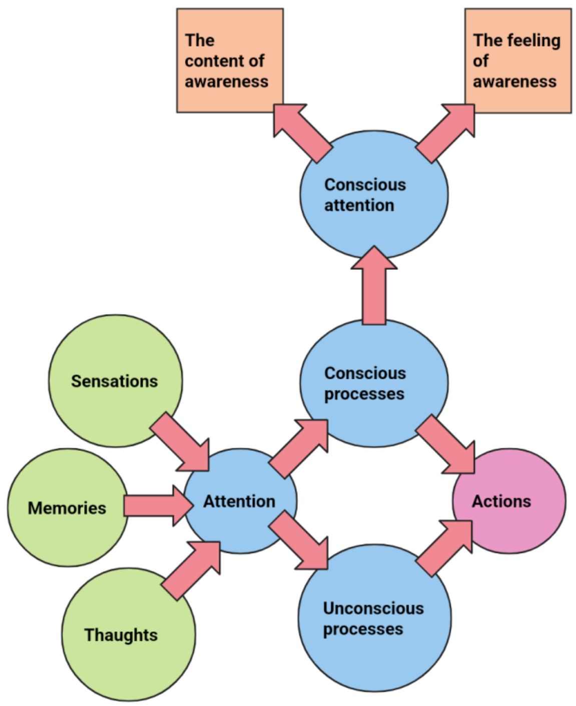

Figure 1. How does attention work? (A suggested map)

Attention acts like a spotlight. It determines what information should be focused on and responded to, and what information should be ignored for what they truly are: unnecessary or not interesting for us. Attention is a mechanism which leads both controlled and automatic processes in order to react to outside stimuli (sensations) and inner stimuli (thoughts and memories) effectively. Attention also plays a role in learning. The mechanism of attention makes learning happen by remembering everything that has been paid more attention to. Imagine you are studying in your room, if your attention is drawn to every outside and inner stimulus and you cannot focus on the material you are reading, memorizing and learning the material will never happen.

2. 3. THE PROBLEM OF ATTENTION

As Allport pointed out: "it seems no more plausible that there should be one unique mechanism, or computational resource, as the casual basis of all attentional phenomena than that there should be a unitary casual basis of thoughts, or perception, or of any other traditional category of folk psychology. . . . Reference to attention (or to the central executive, or even to the anterior attention system) as an unspecified casual mechanism explains nothing." [11].

As mentioned earlier, attention is not a single concept and most of the explanations that have been presented in relation to the mechanism and nature of attention have remained at the level of phenomenological studies. And as Allport also pointed out, explaining attention as a phenomenon that has a single basic mechanism does not add to our knowledge. Although cognitive causal reduction of attention to a single mechanism or several mechanisms has always been of interest. However, the ontological reduction of attention to its components such as conscious and unconscious attention or focused and divided attention can help us understand the nature of attention, more effectively.

3. CONSCIOUSNESS

In many references, the words consciousness and awareness are used as equivalent. In past some psychologists believed that attention and consciousness were the same. But now we know that some of our information processes are done without our conscious awareness. [10].
Consciousness is also not an easy concept to define, and there isn't a specific definition for it. The problem of consciousness, generally referred to as the mind-body problem, which has been the subject of philosophical reflection for thousands of years. [1]. Consciousness is not a single concept it actually is a psychological phenomenon, which is realized by another concept called attention.

3. 1. WHAT IS CONSCIOUSNESS?

The term "consciousness" includes a huge and diverse set of meanings. It is not even obvious that there is any one 'thing' that all uses of the term have in common which could stand as its core referent. [12].
Some dictionaries define consciousness as an ability to be aware of self and surroundings. But these definitions are circular as it has been said, awareness is often used as a synonym of consciousness. [13]. Although, consciousness could not be ontologically reduced to awareness, but it gets us to a good description of consciousness which is: Consciousness allows us to know our own existence (in a subjective way) and the existence of other objects and events (in an objective way).
We wake up each morning to an ongoing stream of conscious events. As we come to consciousness each morning we remember the goals, plans and works we had thought about. That means we can focus our attention on a specific information that will to our consciousness. And that's how we describe being conscious. [14].
You wake up, you open our eyes, and in that moment, the visual world comes alive again. A flow of thoughts, memories and sensation processes begin, You can talk to yourself and in that moment you know that you're awake, you're conscious.
Consciousness includes both the feeling of awareness and the content of awareness. [15]. Feeling of awareness is simply understandable, usually when you are awake you feel aware. But what does it mean by saying the content of awareness? It refers to thoughts, sensations and memories which you can recall at the moment.

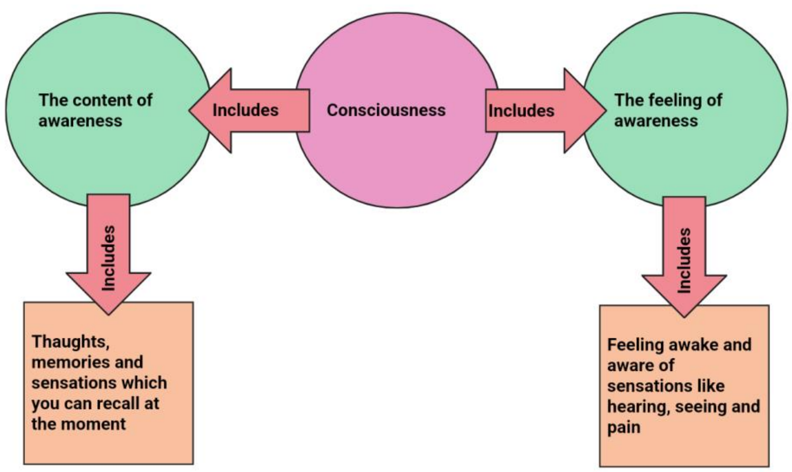

Figure 2. What is consciousness? (A suggested model)

3. 2. THE PROBLEM OF CONSCIOUSNESS

The problem of consciousness, generally referred to as the mind-body problem. Although there have been thousands of years effort to find out what consciousness (or in general, the mind) is, still the nature of consciousness is actually complex and far from clear. [1]. this just one problem of consciousness, which as we said is asking: what's the relationship between mind and body? How does the body shape the mind? But we agreed that body and mind are ontologically entirely distinct, so another question that we shall ask now is: What is consciousness? What are the components of consciousness? This questions lead us to the ontological reduction of consciousness, which is the problem of consciousness we are going to discuss.
As Rowlands pointed out: "Any study of phenomenal consciousness faces an immediate problem. There is no perspicuous way of defining the associated concept. That is, there is no non-circular way of specifying the content of the concept of phenomenal consciousness that does not rely on concepts that are equally obscure. Attempts to explain its content, accordingly, tend to rely on a number of devices, linguistic and otherwise." [16]. Most definitions of consciousness actually provide a description of consciousness, but they do not give a detailed explanation of the nature of consciousness. These definitions may show us to recognize consciousness better by knowing its apparent characteristics, like experiencing emotions and sensory input, the ability to act and self-awareness but they do not lead us to a complete knowledge of consciousness.

3. 3. CONSCIOUS ATTENTION AND COGNITION

What is the relationship between attention and consciousness? Baars argued that access to consciousness is controlled by attentional mechanisms. In everyday language you might hear a lot of sentences like "look at that" or "listen to that". What that actually happens here is that we "look in order to see" and we "listen in order to hear". As Baars says: “The distinction is between selecting an experience and being conscious of the selected event. In everyday language, the first word of each pair ["look"; "listen"] involves attention; the second word ["see"; "hear"] involves consciousness.” [3, 17]. When you look at something, you are paying attention to it. But if we just be a little more precise, we will find that it is actually our awareness that is drawn towards that stimulus by our attention, as if we become more aware of that stimulus.
Conscious attention plays a casual role in cognition, and serves three purposes: First, it helps us interact with the environment. Second, it realizes the continuity of our memories and links them to the present moment. This makes our experience of the present moment feel more real. Third, it helps us to plan for the future based on what we know of past and what we are processing (thinking of) now. [18]. Another role of conscious attention in cognition that has not been considered is the ability to make decisions. When you make a decision, in fact, with the help of your conscious attention, you make a connection between outside stimuli (sensations) and inner stimuli (thoughts and memories) and in this way you start to make a decision.
As we said earlier, attention includes both conscious and unconscious processes. [9]. Even when we are asleep or we are not conscious, our attention is still active and it is working, that would be unconscious attention which leads our automatic processes to an act. For example, when you are driving to your workplace, you may think that your boss may reprimand you for being late, and this thought attracts your conscious attention, but your unconscious attention is focused on your driving, which is an automatic process in this example, and as soon as someone jumps in front of your car, your unconscious attention by directing your information processing towards an appropriate response makes you quickly put your foot on the brake, even though your conscious attention is involved in something else (in this example; thoughts).

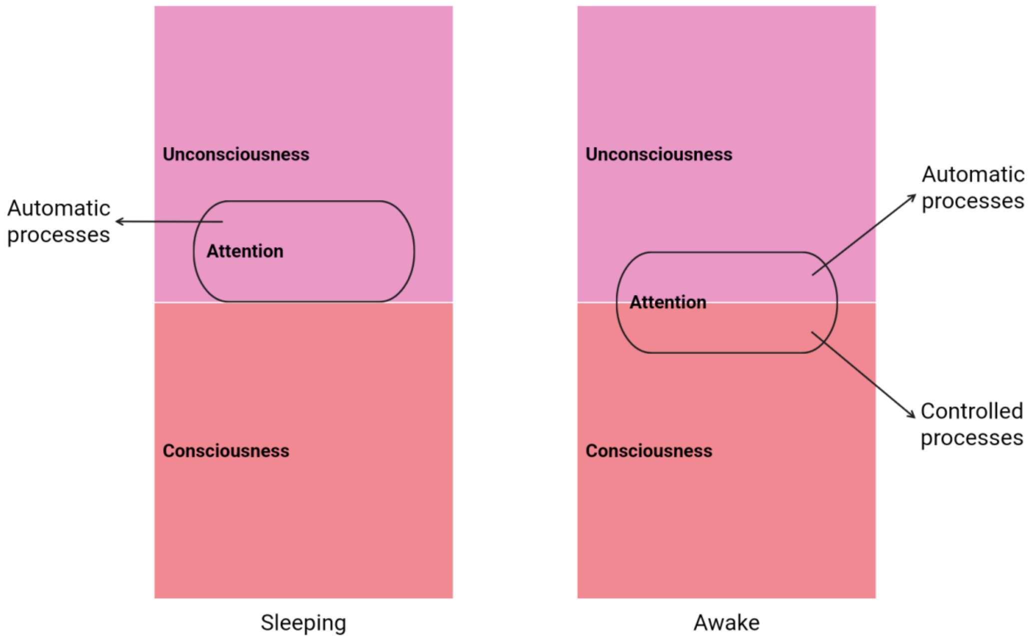

Figure 3. A suggested model to explain the relationship between attention and consciousness: part of attention that overlaps with awareness (consciousness) forms conscious attention

4. CONCLUSIONS

Although the ontological study of attention and consciousness requires a lot of work and has a long way to reach a desired and acceptable result. In this article has been tried to answer these questions: What is attention? How does attention work? What is the right solution for the ontological study of attention? What is consciousness? What is the right solution for the ontological study of consciousness? And what is conscious attention? The purpose of bringing up these questions and trying to answer them was to reach the following conclusions: First, for the ontological study of attention, it should be noted that attention is not a single and simple concept to define and reducing it to a single basic mechanism which is vague and unknowable does not help us. Instead, we should seek to discover the mechanisms of attention and the relationship between them. Second, for the ontological study of consciousness, it should be noted that mind and body are ontologically entirely distinct, as a result for the ontological reduction of consciousness and to know its nature, attention should be studied first, because it has the same mental nature as consciousness.

Third, Conscious attention plays an important role in our cognition, and for this reason, it is very important in cognitive science and cognitive studies should be more focused on it. Learning, remembering, the ability to focus on a specific stimulus, thought or memory, decision-making, planning, personal identity, self-awareness and many other cognitive abilities depend on the presence of conscious attention.

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Decision-making optimization in employee recruitment using fuzzy logic

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Abstract

The application of fuzzy linear programming and solving optimization problems has a very long history. In recent years, the number of people fired from large companies has increased and one of the most important reasons for this issue is the performance of employees and their impact on the progress of organizations. In order to prevent it, it is necessary to calculate the number of employees needed to carry out the desired projects of the company, according to the performance of the employees and the needs of the company and at the same time, considering the optimal number of needed employees as an ideal number before hiring. This value is achievable using optimization and fuzzy linear programming. In this paper, by applying the fuzzy linear programming method, the problem of hiring employees of a factory has been solved. And according to the findings, hiring more people does not necessarily increase performance.

Keywords

optimization, fuzzy logic, fuzzy linear programming, fuzzy simplex, human resources

1. Introduction

In the hiring process, various factors such as the organization's needs, the attitudes and skills of the applicant, the applicant's desires, the budget and desired salary are influential. To make the best decision, all effective parameters should be considered and the most suitable person should be hired to perform the required task, free from bias and discrimination. In the interview process, the individual's qualifications can be practically assessed.

In this article, an attempt has been made to determine the number of employees to be hired for a hypothetical company based on hiring constraints. These individuals have been selected from a factory to find the best solution using the available data from the factory. It should be noted that important parameters such as performance and methods of operation have been taken into account in this selection. The factory has classified the employees into five categories of work, and it is assumed that at least one and up to ten individuals from each team should be selected to ensure the best performance and prevent discrimination. Therefore, ultimately 50 individuals need to be selected. The goal is to obtain the optimal solution for the number of employees to be hired for each team and the overall performance of the selected group.

2. Dataset

The dataset used in this paper is a dataset from Kaggle website called "factory workers' daily performance & attrition" which discusses the performance and impact of employees in a factory, including employees and supervisors. This dataset includes 18 months of daily performance data for 508 employees in a factory. The dataset has various quantitative and qualitative features of employees such as name, age, gender, team, role, level of commitment, level of power, level of health, agility, perception, intellect, and working style. The most important feature of this dataset is having the necessary information to find the relationship between each person's traits with their performance and ultimately their employment in another role for this factory or any other ones. This dataset contains 411948 observations of employees, which makes it even more valuable due to the diverse features included. Due to the large number of features in the dataset, it has been decided to consider only a limited number of features for assessment in this paper. These are the features that have the highest impact on performance and, therefore, the employment of each employee. The most important features of employees in their employment are their work style, power, health, commitment, and performance. Therefore, these features have been selected over other features. These features are qualitative, but since precise results require measurement, relative numerical values have been considered for each of these features. These values are in the range of 0 to 4.1 for performance and 0 to 1 for other features. The work style of employees is also divided into five different categories, named from team A to team E.

3. Methodology

The Pulp library in Python programming language is used to find the optimal solution for determining the number of people required for employment. This library is used for solving linear programming problems and has modules such as LpMaximize for solving maximization problems, LpProblem for defining the linear programming problem, LpStatus for displaying the status of the obtained solution, LpVariable for displaying the optimal variables that create the solution, and so on.

To define the problem, it is necessary to create the needed coefficients for the linear programming model. This is done by taking the average of each of the desired features (strength, health, commitment, and performance) separately for each team. This problem looks for the answer to determination the number of people required for employment according to the factory's conditions. Therefore, the variables of this problem are the number of employees in each team, and it should be noted that at least one person from each team must be employed because the factory needs a complete team of experts in each and every field. The output of this problem and the objective function determines the performance level, and the coefficients of the constraints are the average of the different features. Table1 shows the average features for each team.

Table1- average of each feature

Performance (average)	Strength (average)	Commitment (average)	Health (average)	Team (work style)
0.737041517	0.755292414	0.723606356	1.109482317	A
0.715954913	0.755481072	0.739077414	1.278519779	B
0.771131777	0.744176494	0.747653344	0.926157499	C
0.75674884	0.743358469	0.742491299	0.778422274	D
0.705765077	0.755291629	0.753338434	1.085328533	E

A fuzzy linear programming model is created based on Table1. The linear programming model is defined as the following system:

$$\tilde{C}_1 x_1 \oplus \tilde{C}_2 x_2 \oplus \ldots \oplus \tilde{C}_n x_n \rightarrow M\tilde{a}x$$

Subject to $$\tilde{A}_{i1} x_1 \oplus \tilde{A}_{i2} x_2 \oplus \ldots \oplus \tilde{A}_{in} x_n \tilde{\leq} \tilde{B}_i , i = 1, \ldots, m$$,

$$x_1, x_2, \ldots, x_n \geq 0$$

In which

$$\tilde{A}_{ij}, \tilde{B}_i, \tilde{C}_j, i = 1, \ldots, m; j = 1, \ldots, n$$

Fuzzy sets are a part of real numbers, and the symbol $\oplus$ represents the extended sum. To solve the model using the simplex method, the problem is rewritten in the following form:

Max $\quad \tilde{Z} = \tilde{C}_B x_B + \tilde{C}_N x_N$

s.t. $\quad Bx_B + Nx_N = b$

$$x_B \geq 0, x_N \geq 0$$

In result

$$x_B + B^{-1}Nx_N = B^{-1}b$$

Thus

$$\tilde{Z} + (\tilde{c}_B B^{-1}N - \tilde{c}_N)x_N = \tilde{c}_B B^{-1}b$$

And with considering

$$x_N = 0$$

We have

$$x_B = B^{-1}b = y_0$$

And

$$\tilde{z} = \tilde{c}_B B^{-1}b$$

So the linear programming problem above, is rewritten as Table2.

Table2-fuzzy simplex model

Basis	x_B	x_N	R.H.S.
$\tilde{z}$	0	$\tilde{z}_N - \tilde{c}_N = \tilde{c}_B B^{-1}N - \tilde{c}_N$	$\tilde{y}_{00} = \tilde{c}_B B^{-1}b$
x_B	1	$Y = B^{-1}N$	$y_0 = B^{-1}b$

Table2 contains all the needed information for solving simplex problem. In order to solve it, it should be taken into consideration that the cost of this problem is

$$\tilde{y}_0^T = \tilde{c}_B B^{-1} A - \tilde{c}$$

That

$$\tilde{y}_{0j} = \tilde{c}_B B^{-1} a_j - \tilde{c}_j = \tilde{z}_j - \tilde{c}_j , 1 \leq j \leq n$$

With considering

$$\tilde{y}_{0j} = 0$$

as

$$j = B_i , 1 \leq i \leq m$$

Which for reaching the optimal solution should be

$$\tilde{y}_{0j} \geq 0$$

And apply to all

$$j \neq B_i , 1 \leq i \leq m$$

But if

$$\tilde{y}_{0k} = \tilde{z}_k - \tilde{c}_k \prec 0$$

And

$$\tilde{y}_{0k} = \tilde{z}_k - \tilde{c}_k \prec 0$$

The solution space for the problem is infinite thus, no solution exists. But if k exists as

$$y_{0k} = \tilde{z}_k - \tilde{c}_k \prec 0$$

And B_i exists as

$$y_{ik} \succ 0$$

Then a pivot row of r is achieved by considering

$$\frac{y_{r0}}{y_{rk}} = \min_{1 \leq i \leq m} \{ \frac{y_{i0}}{y_{ik}} \mid y_{ik} \succ 0 \}$$

Which is determinative for the next steps until the condition of feasibility of the problem is satisfied. With reaching this step of the problem, the optimal solution is attainable.

In order to solve the problem using programming, the model and the variables should be defined as

```
model = LpProblem(name="Workers' Daily Performance Optimization With 5 Teams",sense=LpMaximize)

x1 = LpVariable(name="A", lowBound=1, upBound = 10)

x2 = LpVariable(name="B", lowBound=1, upBound = 10)

x3 = LpVariable(name="C", lowBound=1, upBound = 10)

x4 = LpVariable(name="D", lowBound=1, upBound = 10)

x5 = LpVariable(name="E", lowBound=1, upBound = 10)
```

and by considering each of the constraints with respect to the conditions of the problem, they will be defined as

```
model += (0.737*x1 + 0.715*x2 + 0.771*x3 + 0.756*x4 + 0.705*x5 >=10, "health constraint")

model += (0.755*x1 + 0.755*x2 + 0.744*x3 + 0.743*x4 + 0.755*x5 >=20, "commitment constraint")

model += (0.723*x1 + 0.739*x2 + 0.747*x3 + 0.742*x4 + 0.753*x5 <=20, "strength constraint")
```

Then the objective function is acquired with respect to the performance column

```
obj_func = 1.109 * x1 + 1.27*x2 + 0.926*x3 + 0.778*x4 + 1.085*x5

model += obj_func
```

The model is complete at this point. The output of the program is as below

```
Workers'_Daily_Performance_Optimization_With_5_Teams:

MAXIMIZE

1.109*A + 1.27*B + 0.926*C + 0.778*D + 1.085*E + 0.0

SUBJECT TO

health_constraint: 0.737 A + 0.715 B + 0.771 C + 0.756 D + 0.705 E >= 10

commitment_constraint: 0.755 A + 0.755 B + 0.744 C + 0.743 D + 0.755 E >= 20

strength_constraint: 0.723 A + 0.739 B + 0.747 C + 0.742 D + 0.753 E <= 20

VARIABLES

1 <= A <= 10 Continuous

1 <= B <= 10 Continuous

1 <= C <= 10 Continuous

1 <= D <= 10 Continuous
```

1 <= E <= 10 Continuous

To find the optimal solution for the objective function and the variables some modules of Pulp library have been used. These modules help with determination of the existence of a solution for the problem or the outputs of the objective function and the values of the variables which lead to the optimal solution. These modules have been used as

```
status = model.solve()

print(f"status: {model.status}, {LpStatus[model.status]}")

print(f"objective: {model.objective.value()}")

for var in model.variables():

   print(f"{var.name}: {var.value()}")
```

The output of this piece of code is

```
status: 1, Optimal

objective: 31.100553809499996

A: 10.0

B: 10.0

C: 1.0

D: 1.0

E: 5.1673307
```

3. Findings

The output of the program indicates that using ten people from group A, ten people from group B, one person from groups C and D, and finally five people from group E, which together form a group of twenty-seven people, can achieve the highest possible performance considering the constraints for this problem. The obtained value for performance (objective function) has an abstract definition and only makes sense when compared to other groups, but it indicates the highest number that can be achieved for this problem. The results show that it is not necessary to hire more employees in order to achieve the best performance, and in this problem, instead of fifty people, only twenty-seven people can be selected to achieve optimal performance which reduces the hiring cost for the factory remarkably.

4. Conclusions

In many real-world problems that are formulated by linear programming models, there may be a type of uncertainty in some of the model parameters, and this ambiguity may be probabilistic or explicitly expressed through fuzzy numbers. In this study, the fuzzy linear programming was used to determine the hiring process and the number of employees required for an organization, and although the output of the subject is under special and theoretical conditions, generally it can be concluded that the highest performance and the best solution differ under different conditions. Therefore, it is better to investigate the related factors before making decisions and then achieve the best output by applying them in optimization algorithms. Furthermore, the solution employed in this problem can be useful for human resources managers in the hiring process and the determination of the required number of employees. Additionally, given the qualitative nature of the objective function of this problem, it can be concluded that simplex optimization is not only useful in obtaining useful quantitative optimal outputs, but also for qualitative problems. However, it should be noted that the outputs obtained by this method are closer to reality because it considers different factors to achieve the best solution and by considering probabilities, it reaches a solution closer to the real world than other methods. In this paper, we used a real-world dataset and tried to pick the most useful and related features to solve the problem, and also programming the fuzzy simplex model to solve a linear programming problem which can shorten the path to reach the optimal solution while being accurate at the same time. This model not only works for this dataset, but also works totally fine with other qualitative or quantitative datasets and can help solve other linear programming problems of any sort.

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Topic Modeling: Exploring the Processes, Tools, Challenges and Applications

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Abstract

Topic modeling is a data analysis technique that has become increasingly popular in recent years due to the growing availability of large datasets. It enables researchers to uncover underlying themes and topics within large and complex datasets by identifying patterns and relationships among data points. This article provides an overview of the topic modeling process, including the algorithms used, tools,evaluation metrics, and applications across various fields. It also discusses potential areas for future research, such as the integration of other machine learning techniques and the incorporation of temporal and contextual information. Overall, topic modeling represents a powerful tool for data analysis that has the potential to unlock new insights and understanding in a variety of domains.

Keywords

Topic Modeling, Natural Language Processing, Latent Dirichlet Allocation, Non-negative Matrix Factorization, Latent Semantic Analysis

1. Introduction

In recent years, with the explosion of digital data, extracting insights and relevant information from large volumes of unstructured text data has become increasingly challenging. Topic modeling, a subfield of natural language processing (NLP), uses statistical algorithms to extract latent topics or themes from a collection of documents, making it easier to analyze and interpret complex data. The aim of topic modeling is to identify the most common patterns or themes in unstructured text data, which can be used to extract relevant information [1].

There are two prominent topic modeling algorithms: Latent Dirichlet Allocation (LDA) and Non-negative Matrix Factorization (NMF). LDA assumes that each document is a mix of latent topics, where each topic is a probability distribution over a set of words, while NMF is a matrix factorization technique that assumes each document is a linear combination of a few underlying topics. These algorithms have been widely used in data mining, text analysis, and machine learning to uncover patterns, gather insights, and identify hidden structures in text data [2].

One of the most significant advantages of topic modeling is its versatility across domains, as it facilitates the processing of vast amounts of text data, making it a valuable tool in various fields, including social media analysis, market research, and healthcare. Social media platforms provide a wealth of information, with vast amounts of user-generated content that can be challenging to analyze. Topic modeling makes it possible to extract topics of interest to a brand or a company, making it more efficient for businesses to understand customer preferences, opinions, and perceptions of their products [3]. Market research and advertising campaigns can also benefit a great deal from topic modeling since it allows marketers to identify emerging trends, uncover consumer behavior patterns, and tailor consumer messaging to specific target markets. In healthcare, topic modeling is useful in analyzing electronic medical records to identify patterns of diseases and understand the underlying causes of health issues [3].

Moreover, topic modeling has also been used in analyzing historical texts. For instance, researchers have applied topic modeling techniques to examine patterns within massive archives of digitized newspapers and treatises, revealing previously overlooked aspects of history [1]. An exploration of the CORD-19 Open Research Dataset on COVID-19 showed that topic modeling can play a significant role in gathering insights from multiple research papers related to a specific topic [2], [4].

In conclusion, the indispensability of topic modeling in data analysis and interpretation of unstructured text data has been established. Topic modeling is a promising area of research that provides sophisticated data mining and interpretation methods and is applicable to scholars and practitioners alike. With the exponential growth in digital data, topic modeling will continue to be a valuable tool in uncovering hidden trends and extracting critical insights.

1.1. Relationship between NLP and Topic Modelling

Natural Language Processing (NLP) and Topic Modeling are two related but distinct fields that are often used together to extract useful insights from text data. NLP provides the foundation for topic modeling by applying various techniques to preprocess the raw text and make it machinereadable. Topic modeling, on the other hand, utilizes the preprocessed text data to extract underlying topics that are present in the text corpus [5].

NLP is a field of study that deals with the interaction between humans and machines using natural language. Its primary goal is to enable computers to interpret human language and understand the meaning of the text based on context[4]. NLP utilizes techniques like tokenization, stemming, stop-words removal, part-of-speech tagging, and lemmatization to process and transform raw text into a structured format that can be analyzed [6].

Topic modeling is a statistical algorithm used in NLP to discover the underlying topics in a corpus of text. It enables us to find patterns in a large volume of unstructured data, extract information and insights and reveal hidden themes in the data [5]. Popular topic modeling techniques are Latent Dirichlet Allocation (LDA), Non-negative Matrix Factorization (NMF), and Latent Semantic Analysis (LSA) [7].

The combination of NLP and Topic Modeling can be used to extract insights from various text data sources. For example, in customer feedback analysis, NLP can be used to preprocess customer reviews and feedback into a structured format, and then topic modeling is applied to identify the common issues affecting customer satisfaction[20]. Similarly, in market research, NLP can be utilized to analyze customer comments on social media platforms such as Twitter, blogs, and forums to provide a better understanding of customer needs and preferences [8].

In summary, NLP provides the techniques for text preprocessing, while topic modeling algorithms are used to observe the patterns that exist within the preprocessed text.

1.2. Latent Dirichlet Allocation (LDA)

Latent Dirichlet Allocation (LDA) is one of the most popular algorithms utilized in Topic Modeling. It is a probabilistic model designed to identify the underlying topics present in a collection of documents[8]. The algorithm works by analyzing the frequency of co-occurring words in each document and identifying groups of words that commonly occur together [9]. LDA is based on the concept of a generative model, which means it assumes that there is an underlying set of topics that contribute to each document in a corpus. However, since these topics are not directly observable, the LDA algorithm attempts to infer them by analyzing the distribution of words across the documents in the corpus [10].

The LDA algorithm can be broken down into the following steps:

1. **Preprocessing the text:** This involves converting the raw text data into a suitable format for the LDA model to be applied. Preprocessing typically involves tokenization, stop word removal, stemming, and other techniques that reduce noise and improve efficiency [11].
2. **Constructing a Vocabulary:** The next step is to construct a vocabulary of words that appear in the corpus. Each of these words is assigned a unique identifier, typically an integer [11],[12].
3. **Constructing a Document-Term Matrix:** In this step, each document in the corpus is represented as a vector of word counts. The document-term matrix displays the frequency of the occurrence of each word in the documents [1].
4. **Running the LDA Algorithm:** The LDA algorithm is run on the document-term matrix, and it tries to identify the underlying topics that fit the words in each document in the corpus. The number of topics that LDA output depends upon the number of specified topic title parameter during the model training process [13].
5. **Interpreting the Results:** Finally, the results of the LDA algorithm are interpreted to understand the overarching themes that are present in the corpus [13]. The LDA algorithm is widely used in various fields like market research, social media analysis, scientific literature, and others. The output of the algorithm provides valuable insights into the themes that are present in a corpus of text documents [14].

1.3. Non-negative Matrix Factorization (NMF)

NMF is a popular algorithm used in topic modeling. Given a large collection of documents, the goal of topic modeling is to identify patterns or topics that occur frequently across the documents. In topic modeling using NMF, the matrix V represents the collection of documents, W represents the topic-word matrix, and H represents the document-topic matrix [11]. The basic idea behind using NMF for topic modeling is that each document in the collection can be represented as a linear combination of a few dominant topics. Similarly, each topic can be represented as a linear combination of a few dominant words. By identifying these dominant topics and words, we can understand the underlying themes in the collection of documents [15].

In NMF-based topic modeling, we begin by representing all the documents as a matrix V, where each row represents a different document, and each column represents a different word. Next, we perform NMF on the matrix V to obtain the topic-word matrix W and the document-topic matrix H. The columns of W represent the most common words associated with each topic, and the rows of H represent the degree to which each document belongs to each topic [16].

One of the main advantages of using NMF for topic modeling is that it enforces non-negativity, which can make the resulting topics easier to interpret. Additionally, because it does not assume a Gaussian distribution for the data, it can often handle sparse data more effectively than other algorithms. NMF can also be easily parallelized, allowing for efficient processing of large collections of documents [6].

1.4. LATENT SEMANTIC ANALYSIS (LSA)

Latent Semantic Analysis (LSA) is a mathematical method used in natural language processing for topic modeling. It involves analyzing a set of documents to discover the underlying themes or concepts that they share based on the patterns within the text [1].

LSA works by creating a matrix of all the words in a corpus (a collection of texts) and then reducing the dimensionality of the matrix. This allows the algorithm to identify clusters of words that are likely to be associated with certain topics. Essentially, LSA is looking for patterns in the text that let it group similar words together into topics [15].

LSA has become popular in recent years as a way to help machines learn to understand natural language. It can be used in a variety of applications, including search engines, sentiment analysis, and chatbots. By using LSA to identify the most relevant topics in a set of documents, it's possible to make sense of large amounts of data quickly and efficiently [17].

1.5. TOPIC MODELLING PROCESS

Topic modeling is a process used in natural language processing to identify themes or topics in a large collection of documents. Here is a general overview of the topic modeling process:

1. **Data Preprocessing:** The first step in topic modeling is to preprocess the data by cleaning and preparing the text. This includes removing stop words (common words like ”the” or ”and”), converting all the text to lowercase, and stemming or lemmatizing the words (i.e., reducing them to their root form) [11].
2. **Choosing the Algorithm:** Next, you need to choose a suitable algorithm for topic modeling. Popular algorithms include Latent Dirichlet Allocation (LDA) and Non-Negative Matrix Factorization (NMF) [18].
3. **Choosing the Number of Topics:** Once you have chosen an algorithm, you need to determine the number of topics to identify. This is typically done by trial and error, where you evaluate the quality of the results for different numbers of topics.
4. **Model Fitting:** After choosing the algorithm and the number of topics, you can fit the model to the cleaned text. This creates a mathematical representation of the text that allows the algorithm to identify patterns and themes [8].
5. **Interpreting Results:** Once the model is fitted, you can examine the results to identify the most important topics in the text. This includes looking at the most common words and phrases associated with each topic [11].
6. **Refining the Model:** Finally, you can refine the model by further preprocessing the text or tweaking the algorithm parameters. The goal is to identify the most accurate and meaningful topics that represent the content of the documents [8].

By following this process, you can extract valuable insights from large collections of text data. These insights can help to inform decision-making, improve customer experience or drive business growth.

1.6. EVALUATION METRICS FOR TOPIC MODELING

There are several evaluation metrics used for evaluating the performance of topic modeling algorithms. Some of the commonly used ones include:

1. Coherence score: Coherence score measures the degree of semantic coherence between the most probable words in a topic. It measures how well a topic represents a specific concept. Equation 1 shows how to calculate this evaluation parameter.

$$Coherence = \frac{\sum_{i,j=1}^{M} sim(w_i, w_j)}{M(M-1)} \quad (1)$$

In this formula, M represents the number of unique words in the corpus, and sim(wi, wj) represents the similarity score between word i and word j, computed using a certain algorithm, such as pairwise probability or pointwise mutual information.

2. Perplexity: Perplexity calculates the degree of uncertainty or unpredictability of the model. It measures how well the model predicts the words within the dataset. Equation 2 shows how to calculate this evaluation parameter.

$$perplexity(D) = \exp\left(-\frac{\sum_{d \in D} \log p(d)}{\sum_{d \in D} N_d}\right) \quad (2)$$

In this formula, the perplexity of the topic modeling model on a given dataset D is being computed. The formula involves the logarithm of the likelihood of the data, divided by the total number of words in the data. The purpose of the formula is to evaluate how well the model is able to predict the words in the training dataset, and lower perplexity scores mean the model is doing a better job.

3. Topic separation and topic uniqueness: Topic separation and uniqueness evaluate the extent to which topics are distinct and do not overlap with each other. Equations 3 and 4 shows how to calculate this evaluation parameter.

$$TS = \frac{1}{k(k-1)} \sum_{i=1}^{k} \sum_{j=1, j \neq i}^{k} \cos(\theta_i, \theta_j) \quad (3)$$

In this formula, k represents the number of topics, and i and j represent the topic vectors for topic i and topic j, respectively. The cosine similarity score is used to measure the similarity between the two vectors, and the average across all possible pairs of topics is computed to obtain the topic separation score. For Topic Uniqueness:

$$TU = \frac{1}{k}\sum_{i=1}^{k} max_{j=1,j\neq i}^{k} \cos(\theta_i, \theta_j)$$

(4)

In this formula, k represents the number of topics, and i and j represent the topic vectors for topic i and topic j, respectively. The cosine similarity score is used to measure the similarity between the two vectors, and the maximum similarity score between each topic and any other topic is computed and averaged across all topics to obtain the topic uniqueness score.

4. Coverage: Coverage measures the number of documents that a topic is relevant to, indicating whether the topic is present in a sufficient number of documents. Equation 5 shows how to calculate this evaluation parameter.

$$Coverage = \frac{1}{|D|}\sum_{d\in D}\frac{number\ of\ topics\ covering\ the\ document\ d}{total\ number\ of\ topics\ in\ the\ model}$$

(5)

In this formula, D represents the set of all documents in the dataset being analyzed. For each document d in the
set, the formula computes the number of topics that cover the document, divided by the total number of topics in the model. This is then averaged across all documents in the set to obtain the coverage score.

5. Topic distribution: Topic distribution measures the degree of overlap between topics and the extent to which they are distributed evenly across the document set. Equation 6 shows how to calculate this evaluation parameter.

$$P(w|z,\emptyset) = \sum_{t=1}^{T} \emptyset_{z,t}.P(w|\beta_t)$$

(
6)

In this formula, w represents a word in the vocabulary, and z represents a topic in the set of topics being modeled. The parameter zt denotes the probability of topic z generating a word from the topic-specific distribution t. The formula computes the probability of observing word w in the context of topic z by summing over all possible topic-specific distributions t and the corresponding probability zt for each topic.

Each evaluation metric has its advantages and limitations. The choice of an evaluation metric depends on the research problem and the goals of the researcher. To improve the performance of topic modeling algorithms, it is important to consider various evaluation metrics simultaneously and finetune the models accordingly.

2. TOPIC MODELLING TOOLS

There are several tools available for topic modeling. Some of them are:

- **Gensim:**
  Gensim is an open-source and free Python library that specializes in natural language processing (NLP) and topic modeling. It is built on Python programming language and is designed to help developers and researchers solve various NLP challenges. Gensim is intended to automate some of the most tedious, time-consuming, and complex aspects of NLP processing, allowing practitioners to focus on higher-level tasks [12], [19]. Gensim's key feature is its wide range of algorithms for topic modeling, including Latent Dirichlet Allocation (LDA), Hierarchical Dirichlet Process (HDP), and Random Projections. These algorithms allow users to identify latent topics from large datasets of unstructured text data. The library also includes a variety of other features for working with natural language data, such as text preprocessing, word vector embedding, similarity queries, document clustering, and feature extraction [19]. One of the main advantages of Gensim is its scalability, allowing it to handle large datasets with relative ease. It can be used to process corpora with millions of documents and hundreds of thousands of unique features. Additionally, Gensim is built using a modular architecture, which enables it to be easily extended with custom algorithms or data sources [6].
  Another advantage of Gensim is its user-friendly API, which simplifies the implementation of complex NLP processes. The library's API is well-documented and includes a variety of examples and tutorials, making it easy for developers to get started with it. Additionally, Gensim is designed to integrate seamlessly with other libraries in the Python ecosystem, including scikit-learn, spacy [20], and NLTK [11].
  In conclusion, Gensim is a powerful and versatile tool for natural language processing and topic modeling. Its scalability, modularity, and ease of use make it a popular choice for researchers, developers, and practitioners in various industries. As natural language processing becomes increasingly important in fields such as data science, machine learning, and artificial intelligence, Gensim is likely to remain a valuable resource for those working in these areas. Gensim is available from this source (https://radimrehurek.com/gensim/)

- **Mallet:**
  Mallet (Machine Learning for Language Toolkit) is a Java-based open-source software that is used for natural language processing (NLP) and topic modeling. Mallet is primarily used for large-scale machine learning applications in NLP, such as text classification, document clustering, and topic modeling. It is used by researchers in various fields, including computer science, social sciences, and natural language processing [2], [21]. Mallet provides a command-line interface for running various types of NLP analyses, including topic modeling, document classification, and named entity recognition. It can also be used to preprocess textual data, such as removing stop words or stemming words. Mallet's topic modeling approach is based on the popular Latent Dirichlet Allocation (LDA) algorithm that can identify the hidden topics and their underlying semantic structures within a large corpus of textual data [21]. One of Mallet's most significant advantages is its speed and scalability. Mallet can process millions of documents efficiently and effectively, making it ideal for large-scale text analysis. Mallet provides a robust implementation of LDA, which can handle large-scale topic modeling, as well as other probabilistic models [1]. Mallet also

comes with an easy-to-use API that allows developers to use it directly within their own Java code. It is relatively easy to integrate Mallet's topic modeling capabilities into text processing and search engines [16].

In conclusion, Mallet is a powerful and widely used natural language processing toolkit that enables researchers to analyze large collections of textual data. Its scalability, flexibility, and speed make it an attractive option for large-scale machine learning applications in NLP. The Mallet toolkit is freely available for download under an open-source license, making it accessible to researchers and practitioners in various fields. Mallet is available from this source (http://mallet.cs.umass.edu/topics.php)

- **Stanford Topic Modeling Toolbox [22]:**
  The Stanford Topic Modeling Toolbox (TMT) is a free and open-source software package designed for statistical topic modeling of text. It is written in Java and supports various algorithms for topic modeling, including Latent Dirichlet Allocation (LDA), Probabilistic Latent Semantic Analysis (PLSA), and Hierarchical Dirichlet Process (HDP). The TMT is created by the Stanford Natural Language Processing Group. One of the key features of the TMT is its flexibility and extensibility. It allows researchers to experiment with different topic modeling algorithms, preprocessing techniques, and evaluation methods. The toolkit includes comprehensive documentation and tutorials to help researchers get started with their research. The TMT's algorithms are designed to scale effectively to large datasets and be robust to noisy data. Additionally, the software has been optimized for use on a cluster of computers, which means that it can support parallel processing to speed up the analysis process.
  The TMT also provides an interactive topic modeling workspace, which allows researchers to explore their data quickly and easily. The workspace includes visualization tools, such as word clouds and topic dendrograms, and
  allows for the rapid comparison of different models. Overall, the Stanford Topic Modeling Toolbox is a powerful and flexible tool for natural language processing researchers. Its support for various topic modeling algorithms, scalability, and interactive workspace makes it an ideal tool for researchers looking to explore and analyze large collections of textual data. The TMT is freely available for download under an open-source license, making it accessible to researchers and practitioners in various fields. TMT is available from this source (https://nlp.stanford.edu/software/tmt/tmt-0.4/)

- **TM4L [23]:**
  TM4L (Topic Modeling for Learning) is a Python library that is designed to make topic modeling accessible for educators and researchers in the field of education. It is created by the Center for Technology in Learning at SRI International and is freely available under an open-source license. TM4L is primarily designed to assist researchers in identifying themes and topics that may be present in text-based learning data. The library includes a variety of features for text preprocessing, topic modeling, and topic interpretation. The library supports several topic modeling algorithms, including Latent Dirichlet Allocation (LDA) and Nonnegative Matrix Factorization (NMF) [24].
  TM4L also includes interactive visualization tools, including heatmaps and dendrograms, that allow researchers to explore and analyze their data quickly and easily. The visualization tools

enable researchers to identify patterns and trends in their textual data, providing them with valuable insights into the learning process [11].

One of the most significant advantages of TM4L is its focus on the education-specific context. The library contains several features that are specifically designed for the analysis of educational datasets, such as its support for pre-processing of educational data, its ability to make sense of complex, multi-modal data, and its flexible and configurable settings [14].

In conclusion, TM4L is a powerful and accessible tool for educators and researchers who want to perform topic modeling on text-based educational data. Its focus on the education-specific context, interactive visualization tools, and support for multiple algorithms make it an attractive option for those looking to analyze textual data related to learning. TM4L is freely available, including under MIT and GPL-3.0 license, and can be installed via pip or conda, making it easily accessible for research in various fields. TM4L is available from this source (https://tm4l.github.io/)

3. TYPES OF TOPIC MODELLING APPLICATIONS

There are several types of topic modelling applications, including:

1. Information retrieval: Topic modelling can be used in information retrieval systems to improve the accuracy of search results [11].

2. Sentiment analysis: Topic modelling can be used in sentiment analysis to identify the sentiment of a particular document or text [1].

3. Recommender systems: Topic modelling can be used in recommender systems to suggest items to users based on their preferences [2].

4. Market research: Topic modelling can be used in market research to identify trends and patterns in consumer behavior [23].

5. Text classification: Topic modelling can be used in text classification to automatically categorize text documents into predefined classes or categories [11].

6. Language translation: Topic modelling can be used in language translation to improve the accuracy of machine translation systems [12].

7. Content generation: Topic modelling can be used in content generation to automatically generate text that follows a particular style or theme [12].

8. Fraud detection: Topic modelling can be used in fraud detection systems to identify patterns of fraudulent behavior [9].

9. Medical diagnosis: Topic modelling can be used in medical diagnosis to identify relationships between diseases and symptoms [15].

10. Image and video analysis: Topic modelling can be used in image and video analysis to automatically classify images or segments based on their content [15].

4. TYPES OF TOPIC MODELLING CHALLENGES

Topic modeling, the task of identifying the themes or topics within a given corpus of text, is a challenging process that has many obstacles. Some of the types of topic modeling challenges include:

1. Ambiguity: The meaning of words can be ambiguous, which makes it difficult to classify them into specific topics. Some words have multiple meanings, and depending on the context, they can fit into different topics [15].

2. Noise: Text data often contains irrelevant or noisy information that can make it difficult to extract meaningful insights. For instance, spelling mistakes, typographical errors, and random strings of characters can pollute the data [8].

3. Scale: Topic modeling challenges increase as the size of the dataset grows. Large corpora require more computational resources and take longer to process, increasing the complexity of the task [18].

4. Sparsity: In many datasets, the frequency distribution of words is highly skewed, with only a few words occurring several times while most words only appear once or twice. Sparse data makes it difficult to identify coherent topics [23].

5. Domain Specificity: Topic modeling can also be challenging in domain-specific settings, where the vocabulary and syntax can be different from standard language usage. The specialized language and terms may require additional preprocessing [17].

6. Human Interpretation: The topics and their interpretation are subjective to the person who is analyzing the output of the topic modeling algorithms. Their understanding may differ due to their contextual knowledge or other biases [3].

7. Hierarchical relationships: Topics can have hierarchical relationships, where they are composed of sub-topics, and this can be difficult to capture in a flat topic model. Identifying such relationships can lead to more informative results [3].

8. Time Sensitivity: Topic Models are built filter the data irrespective of the timestamp when they have been collected. Hence, as the data continues to grow, topic modeling algorithms need to be adapted or changed to discover new topics that may emerge over time [3].

9. Multimedia Content: Applying topic modeling algorithms to multimedia content such as images, videos, or speech requires specialized techniques and hence involves further topic modeling challenges [6].

10. Ethical and Societal Concerns: The use of topic models to analyze people's sentiments, opinions, or personal data can have severe ethical and societal implications that need to be closely monitored and regulated [9].

5. CONCLUSION

In conclusion, topic modeling is a powerful tool for analyzing large and complex datasets, enabling researchers to uncover underlying themes and topics that may not be apparent through traditional research methods. It has wide-ranging applications, including in fields such as social media analysis, marketing research, and medical research. While there are challenges involved in developing accurate and efficient topic modeling algorithms, ongoing research is focused on refining these models and incorporating additional features, such as contextual information and temporal data. As the field of data science continues to evolve, it is expected that the use of topic modeling will become increasingly widespread and sophisticated, leading to new insights and discoveries in a variety of domains.

6. FUTURE WORK

here are several potential future areas of research for topic modeling:

- **Integration of other machine learning techniques:** Topic modeling can be combined with other machine learning methods like deep learning and reinforcement learning to enhance its capabilities [20].
- **Incorporation of context:** The incorporation of context can help to better understand the relationship between topics and uncover more nuanced patterns in data [18].
- **Exploring cross-lingual topic modeling:** Cross-lingual topic modeling can be used to find similar concepts across different languages [7].
- **Semi-supervised and active learning:** These techniques can help improve the accuracy and efficiency of topic modeling algorithms by using human feedback to refine the models [11].
- **Incorporation of temporal information:** Temporal information can help to better understand the evolution and dynamics of topics over time [16].
- **Evaluation metrics**: Development of evaluation metrics specifically tailored for topic modeling can help to better assess the performance of models [11].
- **Applications of topic modeling:** Topic modeling has potential applications in a variety of fields, including social media analysis, marketing research, and medical research. Future research may focus on developing and fine-tuning topic models for specific applications [12].

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DATA SCIENCE: EXPLORING the JOB ROLES, TOOLS, COMPONENTS, APPLICATIONS and Life Cycle

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Abstract

Data Science is an interdisciplinary field that involves the use of statistical and computational techniques to extract insights and knowledge from complex data. It involves a combination of skills from various fields such as statistics, computer science, and domain-specific knowledge to make sense of large and complex data sets. The field of data science has grown rapidly in recent years due to the availability of big data and the increasing demand for data-driven decision-making in various industries. The process of data science involves data collection, data cleaning, data analysis, and the application of machine learning algorithms to build predictive models. Data science has the potential to transform the way we make decisions in various fields such as healthcare, finance, marketing, and transportation by providing new insights and predictions that were not possible with traditional methods. In this paper, we are giving an introduction to data science, with data science job roles, tools for data science, components of data science, application, etc.

Keywords

Data Science, Machine Learning, Data Analyst, Data Visualization, Model Building

1. Introduction

Data science is a rapidly growing field that combines elements of computer science, statistics, and domain knowledge to extract insights and knowledge from complex data sets [23]. The field is driven by a variety of trends, including the rise of big data, the increasing availability of powerful machine learning algorithms, and the growing need for businesses and organizations to make data-driven decisions [1,2]. At its core, data science is all about extracting meaning from data. This can involve analyzing large data sets to identify patterns and trends, building predictive models that can help forecast future events, or creating visualizations that help communicate complex data in a clear and meaningful way [3]. Data science is a highly interdisciplinary field, which means that practitioners must be comfortable working with a wide range of tools and technologies [4]. This might include specialized programming languages like R or Python, statistical software packages like SPSS or SAS, or big data platforms like Hadoop or Spark [24,5].

In addition to technical skills, data scientists must also possess a range of soft skills that enable them to work effectively with stakeholders across an organization [4]. This might include strong communication skills, the ability to work collaboratively with others, and a deep understanding of the business or organizational context in which they are working [2].

One of the most exciting aspects of data science is its potential to transform a wide range of industries and fields [1]. For example, in healthcare, data scientists are working to develop predictive models that can help identify patients who are at high risk of developing chronic diseases [6]. In finance, data scientists are building algorithms that can help detect fraud and improve risk management. And in transportation, data scientists are using real-time data to optimize traffic management and reduce congestion on our roads [7].

Despite its many benefits, data science poses a number of challenges as well. For one thing, the sheer volume of data that is generated each day can be overwhelming, making it difficult to separate signal from noise [4]. Additionally, data science projects often require significant investments in time, money, and resources, which can pose significant challenges for smaller organizations or those with limited budgets [7]. In order to be successful in data science, therefore, it is important to have a clear understanding of the challenges and opportunities that the field presents [8]. This requires a deep understanding of the underlying technologies and tools that data scientists use, as well as a keen appreciation for the broader societal and economic trends that are driving the field forward [2].

Ultimately, the goal of data science is to make better decisions using data. This might involve identifying new business opportunities, improving operational efficiency, or developing new products and services. Whatever the specific application, data science has the potential to help organizations of all sizes and types unlock new insights and achieve their goals in exciting and transformative ways. In this paper, we are giving an introduction to data science, with data science Job roles, tools for data science, components of data science, application, etc.

1.1 Need for Data Science

The need for data science has never been more pressing. With more data being generated than ever before, organizations across a wide range of industries are recognizing the need to harness this data to drive insights and make better decisions [2]. From healthcare and finance to transportation and retail, data science is transforming the way businesses operate and succeed in a rapidly changing world [4]. One of the key drivers of the need for data science is the rise of big data. Every day, massive amounts of data are generated by a wide range of sources, from social media and mobile devices to sensors and IoT devices [5]. This data is incredibly valuable, but it can also be overwhelming, making it difficult for organizations to effectively manage and derive value from it [3]. This is where data science comes in. Data scientists are experts in working with large and complex data sets, using a range of statistical and machine learning techniques to extract insights and identify patterns that might otherwise be missed [8]. By leveraging these insights, organizations can make better decisions about everything from product development and marketing to customer service and supply chain management.

Another key driver of the need for data science is the increasing availability of powerful machine learning algorithms [3]. These algorithms are able to analyze large and complex data sets in ways that were previously impossible, providing organizations with a range of new tools and techniques for driving insights and making predictions [5].

Machine learning algorithms are particularly valuable for tasks like predictive analytics, which involves using historical data to forecast future events. This can be incredibly useful for everything from predicting customer behavior and demand to identifying potential risks and opportunities for an organization [6]. At the same time, the need for data science is also being driven by the growing demand for data-driven decision making. In today's fast-paced business environment, organizations need to be able to quickly and effectively make decisions based on real-time data. This requires not only the ability to collect and analyze data, but also the ability to communicate insights and recommendations to stakeholders across the organization [3].

Data science plays a critical role in this process, providing organizations with the tools and expertise needed to turn data into actionable insights that can drive better decision making [9]. This might involve creating visualizations that help communicate complex data in a clear and meaningful way, or building predictive models that can help forecast future events and guide strategic planning [4].

Finally, the need for data science is being driven by the increasing importance of data privacy and security. As more data is generated and shared, the risks of data breaches and other security threats are also increasing [10]. This requires organizations to be proactive in protecting their data, and to develop strategies for identifying and mitigating potential risks [5]. Data science can play a key role in this process, providing organizations with the tools and techniques needed to identify potential security threats and take steps to prevent them. This might involve using machine learning algorithms to detect patterns and anomalies in data sets, or developing predictive models that can help identify potential risks before they occur [3].

In conclusion, the need for data science is driven by a range of factors, including the rise of big data, the increasing availability of powerful machine learning algorithms, the growing demand for data-driven decision making, and the need to protect data privacy and security. As organizations across a wide range of industries recognize the value of this field, the demand for skilled data scientists is only set to grow, making it an exciting and rewarding career path for those with the expertise and passion for transforming data into insights and action.

1.2 Data science Jobs

here are some popular job titles in data science with full explanations:

1. **Data Analyst:** A data analyst is responsible for collecting, processing, and performing statistical analyses on large datasets to identify trends and insights that can inform business decisions. They are often responsible for creating reports and visualizations that present the data in an understandable way [5].
2. **Data Scientist:** A data scientist uses statistical and machine learning techniques to build predictive models and analyze large datasets to identify trends, patterns, and insights. They are responsible for designing and implementing algorithms that can be used to automate decision-making processes [7].
3. **Business Intelligence Analyst:** A business intelligence analyst analyzes data and reports to provide insights to business stakeholders and decision-makers. They are responsible for creating dashboards and reports that can be used to monitor business performance and inform strategic decisions [11].
4. **Data Engineer:** A data engineer designs, builds, and maintains the infrastructure required to support data science operations. They are responsible for developing and managing data pipelines, data warehouses, and other data storage and processing systems [9].
5. **Machine Learning Engineer:** A machine learning engineer designs and develops machine learning algorithms and models that can be used to automate decision-making processes. They are responsible for training and deploying these models to production environments [11].
6. **Quantitative Analyst:** A quantitative analyst uses statistical and mathematical models to analyze financial and market data to inform investment decisions. They are often employed by financial institutions, hedge funds, and other investment firms [7].
7. **AI Research Scientist:** An AI research scientist conducts research and development in the field of artificial intelligence, including machine learning, natural language processing, and computer vision. They are responsible for developing new algorithms and techniques that can be used to solve complex problems [10].
8. **Data Architect:** A data architect designs and implements data management systems, including data warehouses and data lakes. They are responsible for ensuring that data is organized and stored in a way that is easy to access and use [8].
9. **Big Data Engineer:** A big data engineer designs, builds, and maintains large-scale data processing systems using technologies like Hadoop, Spark, and NoSQL databases. They are responsible for ensuring that these systems are highly scalable and can handle large volumes of data [3].
10. **Data Mining Engineer:** A data mining engineer develops and implements data mining algorithms to extract insights from large datasets. They are responsible for selecting and optimizing algorithms that can identify patterns and relationships in the data [6].
11. **Predictive Modeling Analyst:** A predictive modeling analyst builds predictive models to forecast future trends and behaviors based on historical data. They are responsible for selecting and optimizing algorithms that can be used to make accurate predictions [6].
12. **Data Visualization Specialist:** A data visualization specialist creates visually appealing and interactive data visualizations to help users better understand data and identify patterns. They are responsible for selecting and designing visualizations that can communicate complex information in an understandable way [6].
13. **Data Quality Manager:** A data quality manager ensures data accuracy, completeness, and consistency by establishing and enforcing data quality standards and procedures. They are responsible for developing and implementing processes that ensure that data is of high quality and can be trusted [2].
14. **Data Governance Analyst:** A data governance analyst defines and implements data governance policies and procedures to ensure data security, privacy, and compliance. They are responsible for developing and implementing policies that protect data from unauthorized access, theft, and cyber attacks [2].
15. **Data Security Analyst:** A data security analyst implements security measures to protect data from unauthorized access, theft, and cyber attacks. They are responsible for identifying and mitigating security risks and ensuring that data is stored and transmitted securely [3].

These are just a few examples of the many job titles available in the data science field.

1.3. Prerequisite for Data Science

Data science is an interdisciplinary field that involves the extraction of insights and knowledge from data using a combination of statistical analysis, machine learning, and computer science. Here are some of the prerequisites for data science that you should consider:

- **Mathematics:** A strong foundation in mathematics is essential for data science. You should have a good understanding of calculus, linear algebra, probability, and statistics [12].
- **Programming**: You should be proficient in at least one programming language, such as Python or R. This will enable you to write scripts to gather, clean, and analyze data, as well as to build machine learning models [10].
- **Data Structures and Algorithms**: Understanding basic data structures and algorithms is important in data science because they help you to manipulate, process, and analyze large datasets [5].
- **Data Manipulation Skills:** Data science is all about working with data, so you should have a good understanding of data manipulation techniques such as filtering, aggregating, and joining data [7].
- **Domain Knowledge:** Having domain knowledge in a specific field, such as finance, healthcare, or marketing can give you an edge in data science as it can help you to understand the data better and ask more relevant questions [8].
- **Communication Skills**: It's important to be able to communicate your findings and insights effectively to stakeholders, so good communication skills are crucial [11].

1.4. Difference between BI and Data Science

Certainly! Both Business Intelligence (BI) and Data Science are related to the analysis of data, but they differ in their focus and approach. Here are some key differences between BI and Data Science:

- **Focus:** BI typically focuses on providing insights into past and present data to support operational and strategic decision-making. Data Science, on the other hand, is concerned with predicting future outcomes and discovering new insights from data [12].
- **Data Sources and Volume:** BI typically uses structured data from internal sources, such as databases and spreadsheets, and deals with a smaller volume of data. Data Science, however, often deals with unstructured data from a variety of sources, such as social media, sensors, and weblogs, and works with large volumes of data [9].
- **Tools and Techniques:** BI tools focus on creating reports, dashboards, and visualizations to provide insights into data. Data Science, on the other hand, uses advanced statistical and machine learning techniques to build predictive models and uncover patterns in data [3].
- **Skillset:** BI professionals typically possess skills in data analysis, database management, and reporting. Data Scientists, on the other hand, require advanced skills in data analysis, programming, statistics, machine learning, and domain expertise [8].
- **Business Goals**: BI is usually used to support business operations and optimize performance, while Data Science is often used to develop new products, services, and revenue streams [1].

In summary, BI is focused on analyzing and reporting on past and present data to support decision-making, while Data Science is focused on building predictive models and uncovering insights from large volumes of data to drive innovation and create new business opportunities.

2. Data Science Components

Absolutely! Data Science involves a combination of technical and non-technical components that work together to extract insights and knowledge from data. Here are some of the key components of Data Science (figure 1):

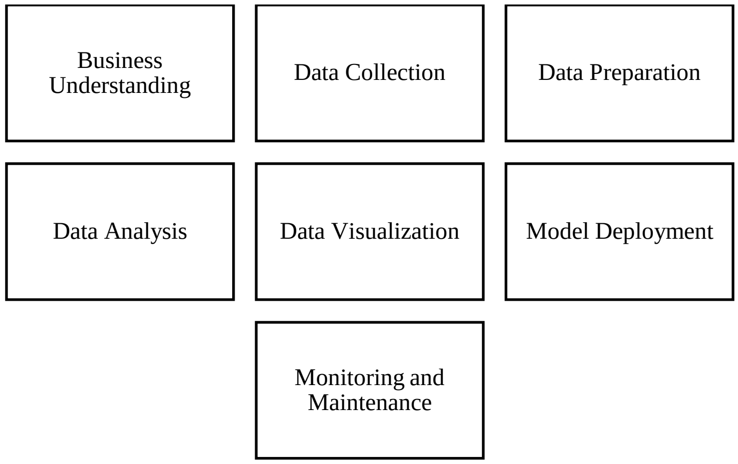

figure 1: Data Science Components

- **Business Understanding**: Before embarking on any Data Science project, it's important to first understand the business problem that you are trying to solve. This involves understanding the business goals, the stakeholders involved, and the data that is available [5].
- **Data Collection:** The next step is to gather the relevant data from various sources, such as databases, APIs, and web scraping. This involves identifying the most important variables and ensuring that the data is clean, complete, and in the right format [6].
- **Data Preparation:** Once the data has been collected, it needs to be prepared for analysis. This involves tasks such as cleaning, transforming, and integrating the data to ensure that it is ready for analysis [7].
- **Data Analysis:** This is where the actual analysis of the data takes place. It involves applying statistical techniques and machine learning algorithms to uncover patterns, relationships, and insights in the data [13].
- **Data Visualization**: Once the analysis has been completed, it's important to communicate the insights effectively to stakeholders. Data visualization techniques such as charts, graphs, and dashboards can be used to convey complex information in a simple and intuitive way [14].
- **Model Deployment**: If a machine learning model has been developed, it needs to be deployed into production to make predictions on new data. This involves integrating the model into a larger system and ensuring that it is scalable, reliable, and secure [8].
- **Monitoring and Maintenance:** Finally, it's important to monitor the performance of the Data Science solution over time and make adjustments as necessary. This involves tracking key metrics, identifying issues, and continually improving the solution [13].

In summary, Data Science involves a combination of technical and non-technical components that work together to extract insights and knowledge from data. These components include business understanding, data collection, data preparation, data analysis, data visualization, model deployment, and monitoring and maintenance.

3. Tools for Data Science

Certainly! Tools for Data Science can be broadly categorized into four categories (figure 2):

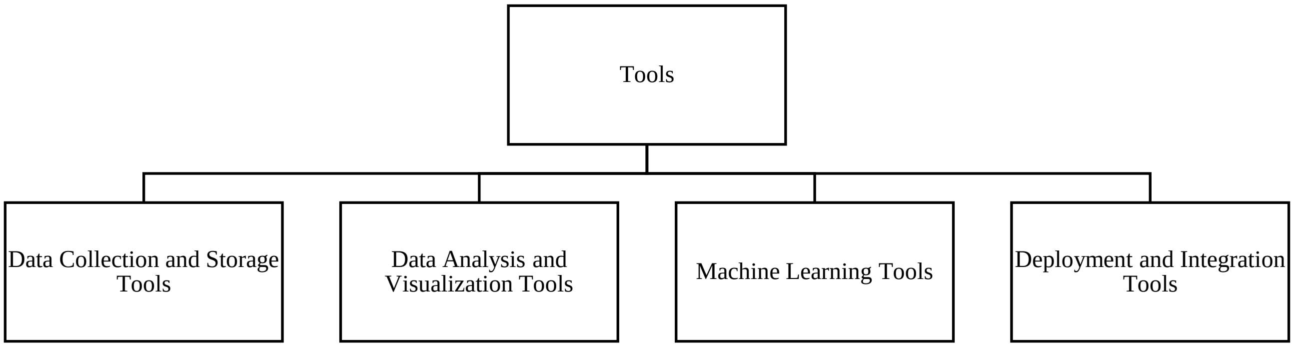

figure 2: Tools for Data Science

- **Data Collection and Storage Tools:** These tools help in collecting, cleaning, and storing data for further analysis. Some popular tools in this category include Apache Hadoop, MongoDB, Apache Cassandra, MySQL, and PostgreSQL [3].
- **Data Analysis and Visualization Tools:** These tools help in analyzing and visualizing the data. They include programming languages like Python and R, as well as visualization tools like Tableau, Power BI, and QlikView [14].
- **Machine Learning Tools:** These tools help in building models and analyzing data patterns. Some popular machine learning tools include scikit-learn, TensorFlow, PyTorch, and Keras [5].
- **Deployment and Integration Tools:** These tools help in deploying and integrating the data science models into production. They include tools like Docker, Jenkins, and Apache Airflow [15].

Each of these tools has its unique features and benefits, and the choice of tool depends on the specific needs of the project. For example, if your project requires storing and managing large amounts of data, Apache Hadoop or MongoDB may be the right choice. If you need to develop a machine learning model, you may choose scikit-learn or PyTorch based on the specific requirements of your project.

4. Machine learning in Data Science

Certainly! Machine learning is a subfield of artificial intelligence that involves teaching computers to learn from data, without being explicitly programmed. In data science, machine learning algorithms analyze data, identify patterns and relationships, and make predictions or decisions based on the patterns identified. There are three main types of machine learning: supervised learning, unsupervised learning, and reinforcement learning [15].

Supervised learning involves training a machine learning model on a labeled dataset, where each data point is associated with a known output or target. The model learns to predict the output for new, unseen data based on the patterns it has learned from the labeled data [12].

Unsupervised learning, on the other hand, involves training a model on an unlabeled dataset, where the output or target is unknown. The goal is to identify patterns and relationships within the data, such as clustering or dimensionality reduction [8].

Reinforcement learning involves training a model through trial and error, where the model receives rewards or penalties based on the actions it takes in an environment. Over time, the model learns to take actions that maximize its rewards and minimize its penalties [3].

Overall, machine learning is a powerful tool for data scientists, allowing them to analyze and make sense of large and complex datasets.

5. Data Science Lifecycle

Data Science Lifecycle is a structured process that data scientists follow to analyze and improve the quality of data-driven insights. It involves several stages that are repeated iteratively until satisfactory results are achieved. The Data Science Lifecycle encompasses the following stages (figure 3):

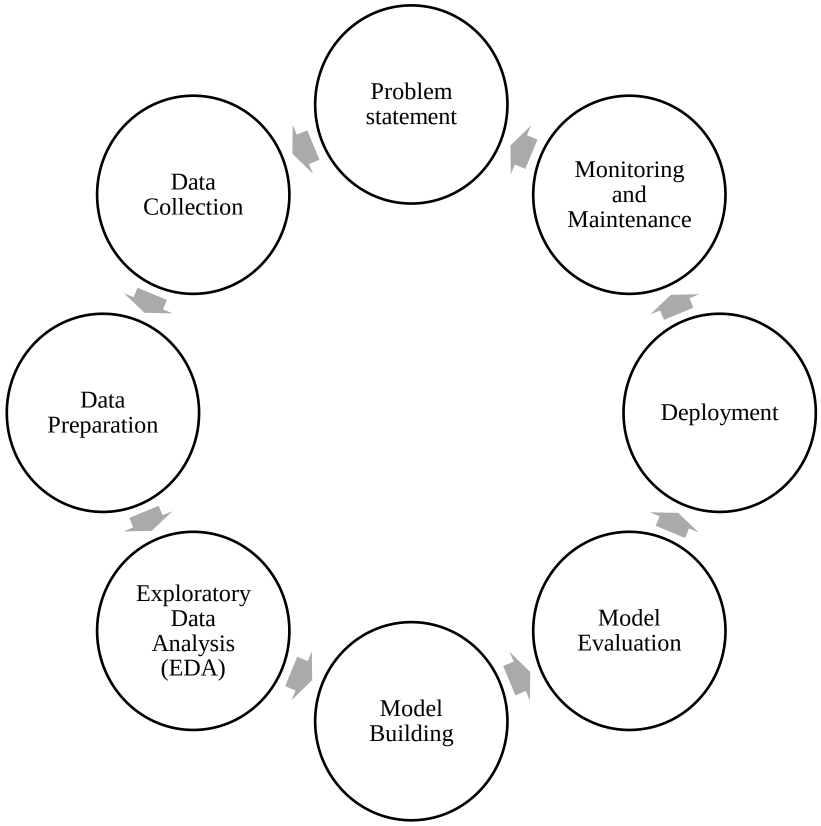

figure 3: Data Science Lifecycle

1. **Problem statement:** Every data science project starts with identifying the business problems or questions that need to be answered. This stage involves defining the scope, objectives, and success criteria for the project [16].
2. **Data Collection:** In the data collection stage, data scientists gather relevant data from various sources, including internal and external databases, APIs, CSV files, and other data sources that are required to answer the business questions [3].
3. **Data Preparation:** This stage involves cleaning, transforming, and filtering the data to make it suitable for analysis. This includes handling missing values, outliers, and data normalization [2].
4. **Exploratory Data Analysis (EDA):** Exploratory data analysis involves visualizing the data to identify patterns, trends, and relationships. This stage helps data scientists to understand the data and identify potential issues that need to be addressed [7].
5. **Model Building:** In this stage, data scientists select the appropriate machine learning algorithms and train the models using the prepared data. They evaluate the performance of the models and fine-tune them for better accuracy [17].

6. **Model Evaluation:** Model evaluation involves validating the performance of the models developed in the previous stage. The accuracy of the models is tested using different metrics and compared with the performance of other models [18].
7. **Deployment:** Once the model is validated and tested, it is deployed into production. This stage involves integrating the model into existing systems and ensuring that it works correctly in a live environment [18].
8. **Monitoring and Maintenance:** In the final stage of the Data Science Lifecycle, the deployed model is monitored continuously to ensure that it continues to provide accurate results over time. Any changes in the data or the business environment are addressed so that the model can be updated as necessary [19].

The Data Science Lifecycle provides a structured approach to data analysis, helping data scientists to identify, create, and deliver valuable insights that can be used to drive business decisions.

6. Applications of Data Science

Data Science has become an integral part of modern businesses and has a wide range of applications across various industries. Here are some of the most common applications of Data Science:

- **Predictive Analytics:** Predictive Analytics is a data science application that involves predicting future events based on historical data. It is widely used in industries such as finance, healthcare, and marketing, where it helps businesses to forecast trends and patterns [16].
- **Fraud Detection:** Fraud detection is one of the most critical applications of data science in the finance and banking sector. Data Science helps detect fraudulent activities by analyzing large volumes of data and spotting any anomalies [5].
- **Recommender Systems:** Recommender systems are used in e-commerce, entertainment, and content platforms to recommend products, movies, and shows based on user preferences. Data Science models analyze user data to understand their preferences and recommend products they are likely to be interested in [20].
- **Natural Language Processing (NLP):** NLP is a subfield of data science that deals with the interaction between computers and human language. It is used in various applications, such as chatbots, sentiment analysis, and speech recognition [21].
- **Image Processing:** Image processing is another application of data science that involves analyzing and interpreting images. It is used in industries such as healthcare, where it helps in diagnosing diseases and detecting abnormalities in medical images [22].
- **Supply Chain Optimization:** Data Science is also used in supply chain management to optimize production and distribution processes. It helps businesses to reduce costs, improve quality, and increase efficiency by analyzing data from various sources [20].
- **Customer Segmentation:** Customer segmentation is a data science application used in marketing to group customers with similar characteristics. It helps businesses to identify and target specific customer segments with personalized marketing campaigns [21].

In summary, Data Science has a wide range of applications, and its use cases are increasing with advancements in technology. It is helping businesses to make data-driven decisions and gain a competitive advantage in their respective industries.

7. Conclusion

In conclusion, Data Science is a rapidly growing and highly important field that has revolutionized businesses across various industries [23]. It involves extracting valuable insights from data using statistical analysis, machine learning, and other techniques. The insights generated by data science help businesses make informed decisions, improve efficiency, and gain a competitive edge.

However, the field of Data Science is continuously evolving, and data scientists must stay up-to-date with the latest tools, technologies, and trends to remain relevant [5]. With the rise of big data, the demand for skilled data scientists has increased exponentially, and this trend is expected to continue in the future.

In today's data-driven world, businesses that leverage the power of data science are more likely to succeed than those that do not. By investing in data science, businesses can gain valuable insights that can help them optimize processes, reduce costs, and improve customer satisfaction.

Overall, Data Science has proven to be a game-changer for businesses, and its applications are only expected to grow in the future. The field offers exciting opportunities for professionals, and those who master the skills required to analyze data will be in high demand for years to come.

In this article, we've introduced science, data science job roles, data science tools, data science components and applications.

9. Future Work

The future of Data Science looks promising, with many new and exciting developments expected to emerge in the field in the coming years. Here are some potential areas of future work for Data Science:

- **Artificial Intelligence:** Data Science and Artificial Intelligence (AI) are closely related fields, and the two are expected to converge in the future. With advances in deep learning, natural language processing, and computer vision, AI is likely to revolutionize the field of Data Science [4].
- **Big Data:** As the volume of data continues to grow exponentially, the field of Data Science will need to develop new techniques and tools to handle Big Data. This will require the development of new algorithms, data storage, and data processing technologies [7].
- **Internet of Things (IoT):** The Internet of Things (IoT) is a rapidly growing field that involves connecting everyday devices to the internet. Data Science is expected to play an important role in IoT by helping to analyze and make sense of the massive amounts of data generated by connected devices [14].
- **Explainable AI:** As AI becomes more prevalent in society, there will be a growing demand for AI models that are transparent and explainable. Data Science will play a critical role in developing explainable AI models that can be trusted by stakeholders [20].
- **Ethical Data Science:** As the use of data becomes more prevalent, there will be a growing need for ethical considerations in data science. Data Scientists will need to ensure that they are using data in a responsible and ethical manner, especially when dealing with sensitive or personal data [21].

In summary, the future of Data Science is bright, with many new and exciting developments expected to emerge in the coming years. By staying up-to-date with the latest advancements in technology and techniques, Data Scientists can continue to make valuable contributions to businesses and society.

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Some Properties of Ultrapowers of Fréchet Algebras

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Abstract

In this paper we consider the notion of ultrapower of Fréchet algebras. Indeed for two Fréchet algebras we prove the canonical map from projective tensor product of their ultrapower into ultrapower of their projective tensor product is injection and has closed image under some conditions.

Keywords

ultrapower, Fréchet algebra, approximation property.

1. Introduction and Preliminaries

Ultrapower of locally convex spaces and Banach spaces was introduced by Heinrich in [6], [7], and [8]. In this paper, we study the construction of the ultrapower of Fréchet algebras. For this purpose, we first show that for a Fréchet algebra $\mathcal{A}$, and a countably incomplete ultrafilter $\mathcal{U}$, its ultrapower $(\mathcal{A})_{\mathcal{U}}$ is Fréchet algebra.

Before giving details of the proof of the above concepts, we begin by recalling some terminology which we generally follow [1, 2, 3, 12]. A complete topological algebra $\mathcal{A}$ whose topology is given by an increasing countable family of sub-multiplicative semi-norms (P_n) is called a Fréchet algebra which we denote it by $(\mathcal{A},(P_n))$.

A filter on a set I is a subset $\mathcal{F}$ of power set I, $P(I)$ such that: (i) empty set is not in $\mathcal{F}$; (ii) $\mathcal{F}$ is closed under finite intersection; (iii) $\mathcal{F}$ is closed under taking supersets. By Zorn's Lemma, the maximal filters exist and are called ultrafilters. Let $\mathcal{F}$ be a filter on a set I and $(x_i)_{i\in I}$ be a family in a topological space. We write $x=\lim_{i\in F} x_i$ if for each open neighborhood U of x, then $\{i\in I : x_i \in U\}\in \mathcal{F}$. We call an ultrafilter $\mathcal{U}$ countably incomplete when there exists a sequence $(U_n)_{n\in N}$ in $\mathcal{U}$ such that $U_1 \supseteq U_2 \supseteq U_3 \cdots$ and $\bigcap_{n\in N} U_n=\varnothing$. In this paper, it is supposed that $\mathcal{U}$ is a countably incomplete ultrafilter.(For more details, see [5, 1.3] and [16]).

Proposition 0.1. [5, Proposition 1.3.4] Let X be a compact topological space, let $\mathcal{U}$ be an ultrafilter on a set I, and let $(x_i)_{i\in I}$ be a family in X. Then there exists $x\in X$ with $x=\lim_{i\in \mathcal{U}} x_i$. Furthermore, if X is Hausdorff, then x is unique.

Note that the converse of the above Proposition is also true.

The rest of this section is devoted to introducing a new kind of ultrafilter, which is called “good ultrafilter". Keisler in 1964, introduced the notion of an $\aleph^+$ -good ultrafilter in the field of sets. He also showed that all countably incomplete ultrafilters are $\aleph_0^+$ -good ultrafilter, where $\aleph_0$ is the first infinite cardinal number.

Before we give the definition of good ultrafilter, we need some notations on functions. Let I be a nonempty set, β a cardinal and $P_\omega(\beta)$ the family of all finite subsets with cardinality less than β. Suppose that f, g are functions from the set $P_\omega(\beta)$ into the power set $P(I)$. We say that $g \le f$ whenever for all $u \in P_\omega(\beta)$, $g(u) \subset f(u)$. Also, g is monotonic whenever for all $u, w \in P_\omega(\beta)$ with $u \subset w$, $g(u) \supset g(w)$. The function g is said to be additive whenever

$$g(u \cup w) = g(u) \cap g(w), \qquad u, w \in P_\omega(\beta).$$

By [2, Lemma 6.1.3] every additive function is monotonic.

Definition 0.2. Let $\aleph$ be an infinite cardinal. An ultrafilter $\mathcal{U}$ on a set I is called $\aleph$ -good ultrafilter if for each cardinal $\beta \le \aleph$ and every monotonic function f from $P_\omega(\beta)$ into $\mathcal{U}$ then there exists an additive function g from $P_\omega(\beta)$ into $\mathcal{U}$ such that $g \le f$.

The next result guarantees that there are sufficiently many good ultrafilters.

Proposition 0.3.[2, Theorem 6.1.4] Let $\aleph$ be an infinite cardinal. On each set of cardinality not smaller than $\aleph$ there exists an $\aleph^+$ -good countably incomplete ultrafilter.

For a locally convex space E let $\aleph_U(E)$ be the least cardinality of a basis of neighborhoods of zero and let $\aleph_B(E)$ be the least cardinality of a fundamental (i.e. cofinal with respect to inclusion) family of bounded sets for which we denote the subset of all absolutely convex closed bounded of it by $B(E)$ and $\aleph(E) = \max(\aleph_U(E), \aleph_B(E))$. Since by [9, Corollary 16.3] every metrizable locally convex space that has a fundamental sequence of bounded sets is normable, for Fréchet space E with $\aleph(E) = \aleph_0$, E is Banach space, where $\aleph_0$ is the first infinite cardinal number.

2. Ultrapower of Fréchet algebras

Let I be an index set equipped with a countably incomplete ultrafilter $\mathcal{U}$. The linear subspace $\ell_\infty(I, \mathcal{A})$ of $\prod_i \mathcal{A}$ for Fréchet algebra $(\mathcal{A}, (P_n))$ can be defined as follows:

$$\ell_\infty(I, \mathcal{A}) := \{(x_i) \in \prod_i \mathcal{A} : \sup_i P_n(x_i) < \infty, \textit{for all } n \in N\}.$$

This space with respect to semi-norms Q_n which are defined as

$$Q_n((x_i)) := \sup_{i \in I} P_n(x_i), \qquad (x_i) \in \ell_\infty(I, \mathcal{A}),$$

is a Fréchet space. Since for any $(x_i) \in \ell_\infty(I, \mathcal{A})$, the set $\{P_n(x_i) : i \in I\}$ is bounded in R, the Proposition 0 implies that $\lim_{\mathcal{U}} P_n(x_i)$ exists and closed subspace $\mathcal{N}_{\mathcal{U}}$ can be defined as follows:

$$\mathcal{N}_{\mathcal{U}} = \{(x_i) \in \ell_\infty(I, \mathcal{A}) : \lim_{\mathcal{U}} P_n(x_i) = 0 \ \ for\ all\ n \in N\}.$$

So the quotient space $(\mathcal{A})_{\mathcal{U}} := \dfrac{\ell_\infty(I, \mathcal{A})}{\mathcal{N}_{\mathcal{U}}}$ is well defined and its elements are denoted by $(x_i)_{\mathcal{U}} := (x_i) + \mathcal{N}_{\mathcal{U}}$ for $(x_i) \in \ell_\infty(I, \mathcal{A})$. For any subset $B \subseteq \mathcal{A}$ we will define $(B)_{\mathcal{U}}$ as $(x_i)_{\mathcal{U}} \in (\mathcal{A})_{\mathcal{U}}$ such that $x_i \in B$.

Using above notations for all $n \in N$ we can define increasing semi-norms Q'_n as follows on $(\mathcal{A})_{\mathcal{U}}$:

$$Q'_n((x_i)_{\mathcal{U}}) := \inf\{Q_n((x_i) + (y_i)) : (y_i) \in \mathcal{N}_{\mathcal{U}}\}. \tag{1}$$

Given semi-norms on $(\mathcal{A})_{\mathcal{U}}$ make this space complete according to [16, Proposition 3.2.7], so $(\mathcal{A})_{\mathcal{U}}$ is Fréchet space. As $\mathcal{A}$ is algebra, for all $(x_i)_{\mathcal{U}}, (y_i)_{\mathcal{U}} \in (\mathcal{A})_{\mathcal{U}}$ the product is defined by $(x_i)_{\mathcal{U}} \cdot (y_i)_{\mathcal{U}} := (x_i y_i)_{\mathcal{U}}$ and $(\mathcal{A})_{\mathcal{U}}$ becomes Fréchet algebra.

Proposition 1.1. Let $\mathcal{A}$ be a Fréchet algebra. Then for every $(x_i)_{\mathcal{U}} \in (\mathcal{A})_{\mathcal{U}}$

$$Q'_n((x_i)_{\mathcal{U}}) = \lim_{\mathcal{U}} P_n(x_i). \tag{2}$$

Proof. Similar to the proof of [11, Theorem 2.4].

Let $(\mathcal{A}, (R_n))$ and $(\mathcal{B}, (S_n))$ be two Fréchet algebras. Then by [10] we can define the projective tensor product $\mathcal{A} \hat{\otimes} \mathcal{B}$ with induced topology by the following sub-multiplicative semi-norms

$$(R_n \otimes S_n)(T) := \inf\{\sum_{i=1}^{k} R_n(x_i) S_n(y_i) : T = \sum_{i=1}^{k} x_i \otimes y_i \in \mathcal{A} \otimes \mathcal{B}\}.$$

And its completion given by

$$(R_n \hat{\otimes} S_n)(T) = \inf\{\sum_{i=1}^{\infty} R_n(x_i) S_n(y_i) : T = \sum_{i=1}^{\infty} x_i \otimes y_i \in \mathcal{A} \hat{\otimes} \mathcal{B}\}.$$

By above discussion we can define $((\mathcal{A} \hat{\otimes} \mathcal{B})_{\mathcal{U}}, (R_n \hat{\otimes} S_n)')$, where $(R_n \hat{\otimes} S_n)'$ is the semi-norm that we obtain from $R_n \hat{\otimes} S_n$ as constructed as above, so $(\mathcal{A} \hat{\otimes} \mathcal{B})_{\mathcal{U}}$ is Fréchet algebra and by Proposition 1 we have:

$$(R_n \hat{\otimes} S_n)'(T_i)_{\mathcal{U}} = \lim_{\mathcal{U}} R_n \hat{\otimes} S_n(T_i), \qquad (T_i)_{\mathcal{U}} \in (\mathcal{A} \hat{\otimes} \mathcal{B})_{\mathcal{U}}.$$

Let $(E,(R_n))$ and $(F,(S_n))$ be two Fréchet spaces. Then we consider the canonical map

$$\psi_0 : (E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}} \to (E \hat{\otimes} F)_{\mathcal{U}},$$

as in [4] by $\psi_0((x_i)_{\mathcal{U}} \otimes (y_i)_{\mathcal{U}}) := (x_i \otimes y_i)_{\mathcal{U}}$, where $(x_i)_{\mathcal{U}} \in (E)_{\mathcal{U}}$, $(y_i)_{\mathcal{U}} \in (F)_{\mathcal{U}}$ and $(x_i \otimes y_i)_{\mathcal{U}} \in (E \hat{\otimes} F)_{\mathcal{U}}$.

Definition 1.2. A locally convex space E has approximation property whenever the space of finite rank operators on E is dense in the space of continuous operators on E with respect to the topology of uniform convergence on all precompact subset of E and we say that it has bounded approximation property (b.a.p in abbreviation) if these finite rank operators are bounded with respect to this topology.

Note. If E is quasi-complete space then the topology of uniform convergence on precompact subsets coincides on topology of uniform convergence on compact subsets, especially in Fréchet spaces. The boundedness of finite rank operators on precompact sets for Fréchet spaces means that these operators are equicontinuous (see [15, Theorem 4.2]). In the other hand the topology of simple convergence and precompact convergence on equicontinuous subset of space of countinuous operators are identical (see [15, Theorem 4.5]) so we can formulate the definition of b.a.p as below for Fréchet spaces:

A Fréchet space $(E,(P_n))$ has b.a.p if for every continuous operator T on E with respect to the topology of simple convergence there is a net $(T_\alpha)_{\alpha\in\Lambda}$ of finite rank operators on E such that for all $n \in N$ there exists a $m \in N$ and $C_n > 0$ that $P_n(T_\alpha(x)) \le C_n P_m(x)$ for all $\alpha \in \Lambda$ and $x \in E$ and for every $\varepsilon > 0$ there exists T_α with $P_n(T_\alpha x - Tx) < \varepsilon$ for every $n \in N$ and $x \in E$.

Lemma 1.3. A locally convex space E with the family of semi-norms $(P_\alpha)_{\alpha\in I}$ has approximation property if and only if for every locally convex space F with the family of semi- norms $(Q_\beta)_{\beta\in J}$, every continuous operator $T : E \to F$, every precompact subset K of E and every $\varepsilon > 0$, there exits a finite rank operator $S : E \to F$ such that $Q_\beta(Tx - Sx) < \varepsilon$ for every $x \in K$ and $\beta \in J$.

Proof. Let E have approximation property and $T : E \to F$ be a continuous operator. Given $\varepsilon > 0$ and precompact subset K of E, there is a finite rank operator U such that

$$P_\alpha(x - Ux) < \varepsilon, \qquad \alpha \in I, x \in K.$$

Since T is continuous, there exists semi norms $\{P_i : i = 1,\cdots,n\}$ and a constant $C > 0$, such that

$$Q_\beta(Tx) \le C \sup_{1\le i\le n} P_i(x), \qquad \beta \in J, x \in E,$$

(see [15, Chapter 3, 1.1]). Now let $TU = S$. Then, S is a finite rank operator from E into F such that

$$Q_\beta(Tx - TUx) < C\sup_{1\le i\le n} P_i(x - Sx) < C\varepsilon,$$

for all $\beta \in J$. So it completes the proof. The reverse is obvious.

Proposition 1.4. Let $(E,(P_n))$ be a Fréchet space with approximation property and let $(F,(Q_n))$ be a Fréchet space. Suppose that for every $\psi \in F'$ and for all $u = \sum_{n=1}^{\infty} x_n \otimes y_n \in E\hat{\otimes} F$ where (x_n) and (y_n) are bounded sequences in E and F respectively, if $\sum_{n=1}^{\infty}\psi(y_n)x_n = 0$ then $u = 0$.

Proof. Let $u \in E\hat{\otimes} F$. Recall that u can be written as a series $u = \sum_{n=1}^{\infty}\lambda_n(x_n \otimes y_n)$, where (x_n), (y_n) are bounded null sequences in E and F respectively and scalar sequence $\{\lambda_n\}$ with $\sum_{n=1}^{\infty}|\lambda_n| < 1$ [10, \$414(6)\$]. Let F_b' and $\mathcal{L}_b(E,F_b')$ be respectively the spaces of linear continuous operators from F into C and E into F_b' with respect to the bounded convergence topology (see [10, 39]). Given $T \in \mathcal{L}_b(E,F_b')$ and $\varepsilon > 0$, since E has the approximation property, for each $\varepsilon > 0$, there exists a finite rank operator $S: E \to F_b'$, such that

$$R_M(Tx - Sx) < \varepsilon, \qquad x \in K = \{x_n : n \in N\} \cup \{0\},$$

where R_M is the semi norm related to locally convex space F_b', is defined by $R_M(f) := \sup_{y\in M}|f(y)|$ for all $f \in F_b'$ and bounded subset M of F. We have $Sx = \sum_{i=1}^{m}\phi_i(x)\psi_i$ where $\phi_i \in E'$ and $\psi_i \in F'$. Take

$$\langle u, S\rangle = \langle \sum_{n=1}^{\infty} x_n' \otimes y_n', S\rangle := \sum_{n=1}^{\infty}(Sx_n')y_n'.$$

Since by [1, Proposition 2] $\mathcal{L}_b(E,F_b') = (E\hat{\otimes} F)'$ where the right hand side has the topology of uniform convergence on the bounded sets of the form $\overline{\Gamma}(C \otimes D)$, such that C and D are bounded subsets in E and F respectively and $\overline{\Gamma}$ is the closed convex hull E and F. In this case, we have

$$\langle u, S\rangle = \sum_{n=1}^{\infty}\sum_{i=1}^{m}\phi_i(x_n')\psi_i(y_n') = \sum_{i=1}^{m}\phi_i(\sum_{n=1}^{\infty}\psi_i(y_n')x_n') = 0.$$

Therefore for bounded subset $M = \{y_n\}$ in F we have

$$|\langle u, T\rangle| \le |\langle u, T - S\rangle| + |\langle u, S\rangle|$$

$$\le \sum_{n=1}^{\infty}|((T - S)\lambda_n x_n)y_n|$$

$$\le \sum_{n=1}^{\infty}|\lambda_n| R_M(Tx_n - Sx_n)$$

$$\leq \sum_{n=1}^{\infty} |\lambda_n| \varepsilon \leq \varepsilon.$$

Since T and ε are arbitrary, it follows that $\langle u, T\rangle = 0$, thus $u = 0$, which is the desired conclusion.

Theorem 1.5. If $(E)_{\mathcal{U}}$ with $\aleph(E)^+$-good ultrafilter has the approximation property, then ψ_0 is an injection for any Fréchet space F.

Proof. Suppose that $T \in ((E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}}, (P_n' \hat{\otimes} Q_n'))$ has a representation $T = \sum_{n=1}^{\infty} x_n \otimes y_n$ with $\sum_{n=1}^{\infty} P'(x_n) Q'(y_n) < \infty$. If $(E)_{\mathcal{U}}$ has the approximation property then, by proposition 1, if $T \in ((E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}}, (P_n' \hat{\otimes} Q_n'))$ is non-zero, then there exist $\mu \in (E)_{\mathcal{U}}'$ and $\lambda \in (F)_{\mathcal{U}}'$ with

$$0 \neq \langle \mu \otimes \lambda, T\rangle = \sum_{n=1}^{\infty} \langle \mu, x_n\rangle \langle \lambda, y_n\rangle.$$

As we only care about the value of μ on the countable set $\{x_n\}$, since $\mathcal{U}$ is $\aleph(E)^+$-good ultrafilter by [6, Theorem 3.4] we may suppose that $\mu \in (E')_{\mathcal{U}}$, and similarly, that $\lambda \in (F')_{\mathcal{U}}$, say $\mu = (\mu_i)$ and $\lambda = (\lambda_i)$. Let representatives $x_n = (x_n^{(i)})$ and $y_n = (y_n^{(i)})$. Then by absolute convergence,

$$\langle \mu \otimes \lambda, T\rangle = \lim_{\mathcal{U}} \sum_{n=1}^{\infty} \langle \mu_i, x_n^{(i)}\rangle \langle \lambda_i, y_n^{(i)}\rangle = \langle (\mu_i \otimes \lambda_i), \psi_0(T)\rangle.$$

Hence we must have $\psi_0(T) \neq 0$ and it completes the proof.

Theorem 1.6. If $(E)_{\mathcal{U}}$ with $\aleph(E)^+$-good ultrafilter has bounded approximation property, then ψ_0 has closed image.

Proof. Take $0 \neq u \in (E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}}$ and suppose that $\sum_{n=1}^{\infty} \alpha_n (x_n \otimes y_n)$ is a representation of u with $x_n \to 0$, $y_n \to 0$ and $\sum_{n=1}^{\infty} |\alpha_n| \leq 1$, see [10, 414(6)]. Let $k = \sum_{n=1}^{\infty} |\alpha_n|$, since by [?, Proposition 22.14] for absolutely convex zero neighborhood V of $(E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}}$ we have Minkovski functional for the set V $\|u\|_V = \sup_{T \in V^\circ} |\langle u, T\rangle|$, without loss of generality we can assume $0 \neq \|u\|_V$ (otherwise we can find a V such that $0 \neq \|u\|_V$) so for $\varepsilon = \dfrac{\|u\|_V}{2(k+1)}$ there exists a $T \in V^\circ$ such that $\dfrac{\|u\|_V (2k+1)}{2(k+1)} = \|u\|_V - \dfrac{\|u\|_V}{2(k+1)} \leq |\langle u, T\rangle|$. Now because $\mathcal{L}_b((E)_{\mathcal{U}}, ((F)_{\mathcal{U}})_b') = ((E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}})'$, by [1, Proposition 2] we can assume $T \in \mathcal{L}_b((E)_{\mathcal{U}}, ((F)_{\mathcal{U}})_b')$ and since $(E)_{\mathcal{U}}$ has approximation property there exists a finite rank operator $S : (E)_{\mathcal{U}} \to (F)_{\mathcal{U}}'$ such that $R_M(Tx - Sx) < \dfrac{\|u\|_V}{2(k+1)}$ for $x \in \{x_n\} \cup \{0\}$ and $M = \{y_n\}$ where R_M is as in Proposition 1. So we have

$$|\langle u, T-S\rangle| \le \sum_{n=1}^{\infty} |((T-S)\alpha_n x_n) y_n|$$

$$\le \sum_{n=1}^{\infty} |\alpha_n| R_M((T-S)x_n)$$

$$\le \varepsilon \sum_{n=1}^{\infty} |\alpha_n|.$$

and it follows that:

$$|\langle u, S\rangle| \ge |\langle u, T\rangle| - \varepsilon \sum_{n=1}^{\infty} |\alpha_n|$$

$$\ge \frac{(2k+1)\|u\|_V}{2(k+1)} - \frac{k\|u\|_V}{2(k+1)} = \frac{\|u\|_V}{2}.$$

Let $\sum_{j=1}^{N} \mu_j \otimes \lambda_j$ be representation of S with $(\mu_j) \subset (E)_{\mathcal{U}}'$ and $(\lambda_j) \subset (F)_{\mathcal{U}}'$ so that $\langle u, S\rangle = \sum_{n=1}^{\infty} \sum_{j=1}^{N} \langle \mu_j, \alpha_n x_n\rangle \langle \lambda_j, y_n\rangle$. By consideration of closed span $\{x_n\}$ and span $\{y_n\}$ and [6, Theorem 3.4], since $\mathcal{U}$ is $\aleph(E)^+$-good ultrafilter we can find operator $G : \overline{span\{\mu_i\}} \to (E')_{\mathcal{U}}$ and $H : \overline{span\{\lambda_i\}} \to (F')_{\mathcal{U}}$ such that $\|H\zeta\|_{(B_1)_{\mathcal{U}}} = \|\zeta\|_{(B_1)_{\mathcal{U}}}$ and $\|G\eta\|_{(B_2)_{\mathcal{U}}} = \|\eta\|_{(B_2)_{\mathcal{U}}}$ where $B_1 \in B(E)$ and $B_2 \in B(F)$ and $\langle a, G\zeta\rangle = \langle a, \zeta\rangle, \quad \langle b, H\eta\rangle = \langle b, \eta\rangle, \quad a \in (E)_{\mathcal{U}}, b \in (F)_{\mathcal{U}}$. In the other hand suppose $R_j^{(1)} : (E')_{\mathcal{U}} \to E'$ and $R_j^{(2)} : (F')_{\mathcal{U}} \to F'$ will be defined by $R_j^{(1)}(\zeta_i)_{\mathcal{U}} = \zeta_j$ and $R_j^{(2)}(\eta_i)_{\mathcal{U}} = \eta_j$ respectively. Now we define $S_j = R_j^{(1)} G \otimes R_j^{(2)} HS$. Take $z = \sum_{i=1}^{n} x_i \otimes y_i \in (E)_{\mathcal{U}} \otimes (F)_{\mathcal{U}}$ and suppose that $R_j^1 G\mu_i = \mu_i^j$ and $R_j^2 H\lambda_i = \lambda_i^j$ then:

$$\langle S, z\rangle = \langle \sum_{k=1}^{n} \mu_k \otimes \lambda_k, \sum_{i=1}^{m} x_i \otimes y_i\rangle = \sum_{k=1}^{n} \sum_{i=1}^{m} \langle \mu_k, x_i\rangle \langle \lambda_k, y_i\rangle$$

$$= \sum_{k=1}^{n} \sum_{i=1}^{m} \langle G\mu_k, x_i\rangle \langle H\lambda_k, y_i\rangle$$

$$= \lim_{\mathcal{U}} \sum_{k=1}^{n} \sum_{i=1}^{m} \langle \mu_k^j, x_i^j\rangle \langle \lambda_k^j, y_i^j\rangle$$

$$= \lim_{\mathcal{U}} \sum_{k=1}^{n} \sum_{i=1}^{m} \langle R_j^1 G\mu_k, x_i^j\rangle \langle R_j^2 H\lambda_k, y_i^j\rangle$$

$$= \lim_{\mathcal{U}} \langle S_j, z_j\rangle \qquad (3)$$

where $z_j = \sum_{i=1}^{m} x_i^j \otimes y_i^j$.

Now let $(\|\cdot\|_{(k)})_{k\in N}$ and $(\|\cdot\|_k)_{k\in N}$ be a subsets of fundamental seminorms on $((E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}})'$ and $(E \hat{\otimes} F)'_{\mathcal{U}}$ respectively which for $\zeta \in ((E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}})'$ and $\eta \in (E \hat{\otimes} F)'$ are defined by:

$$\|\zeta\|_{(k)} = \sup_{x\in U_{(k)}} |\langle x,\zeta\rangle|, \quad \|\eta\|_k = \sup_{x\in U_k} |\langle x,\eta\rangle|$$

where

$$U_{(k)} = \{x \in (E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}} : (P_k)' \hat{\otimes} (Q_k)'(x) \le 1\} \; and \; U_k = \{x \in E \hat{\otimes} F : P_k \hat{\otimes} Q_k(x) \le 1\}$$

and P_k, Q_k are belong to fundamental system of seminorms of E and F respectively. So by (??) and $W = U_k \subset E \otimes F$ we have

$$\|(S_j)\|_W = \|S\|_W \le \|S\|_{(W)_{\mathcal{U}}} \le \|S\|_{(k)}.$$

since $S \in ((E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}})'$ is bounded so there is a constant $C > 0$ such that $\|S\|_{(k)} \le C$ for all $k \in N$ and this C does not depend on choose of S. By a calculation for $u \in (E)_{\mathcal{U}} \otimes (F)_{\mathcal{U}}$ we have $\langle u, S\rangle = \langle (S_j), \psi_0(u)\rangle$ so for all $k \in N$:

$$\frac{\|u\|_V}{2} \le |\langle (S_i), \psi_0(u)| \le \|(S_j)\|_{(k)} \, (P_k \otimes Q_k)'(\psi_0(u)) \le C(P_k \otimes Q_k)'(\psi_0(u)).$$

Hence for all $u \in (E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}}$:

$$\frac{\|u\|_V}{2} \le C(P_k \hat{\otimes} Q_k)'(\psi_0(u)).$$

This implies that if $\{\psi_0(u_n)\}$ is Cauchy sequence then $\{u_n\}$ is Cauchy sequence and since $(E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}}$ is complete it converges to an element u in $(E)_{\mathcal{U}} \hat{\otimes} (F)_{\mathcal{U}}$. Then $\lim_n \psi_0(u_n) = \psi(u)$ and hence this shows that the image of ψ_0 is closed.

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Projective Limit of Flat Fréchet Modules

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Abstract
In this paper we investigate relation between flatness of a Fréchet module and its relative Banach module in its Arens-Michael decomposition.

Keywords
Homology, Flat, Fréchet module, Projective limit.

1. Introduction and Preliminaries

The homology and cohomology of locally convex algebras first appeared in the works of Taylor [4] then Helemskii [2] developed his works based on three important concepts projective, injective and flat modules and found some important relation between these algebraic concepts and functional analysis concepts such as amenability. In this paper we discuss the relationship between Fréchet modules and its component in projective limit. Before, we introduce some definitions and notations.

Definition 1.1. By a projective tensor product $\otimes_\pi$ for l.c.s.' s we shall mean an assignment to each pair (E, F) of Hausdorff l.c.s.'s a Hausdorff 1.c.s. $E \otimes_\pi F$ such that its topology is generated by seminorms

$$p_\alpha \otimes_\pi q_\beta := \inf \sum p_\alpha(x_i) q_\beta(y_i)$$

where (p_α) and (q_β) are the family of seminorms on E and F respectively that generates those topologies and infimum is taken over all algebraic tensor product E and F. we denote the completion of projective tensor product by $\hat{\otimes}_\pi$.

Definition 1.2. An algebra (relative to $\otimes_\pi$) will be a Hausdorff l.c.s., A, with an associative multiplication and an identity, such that the map $a \times b \to ab : A \times A \to A$ extends to a continuous linear map from $A \otimes_\pi A$ to A.

Definition 1.3. If A is an algebra (relative to $\otimes_\pi$) then a left A-module (relative to $\otimes_\pi$) is a Hausdorff 1.c.s. E together with an associative module operation $(a, x) \to ax : A \times E \to E$ which extends to a continuous linear map $A \otimes_\pi E \to E$. In addition we require that $1x = x$, where 1 is the identity of A. Right modules are defined analogously. An A-bimodule is an 1.c.s. with both a left A-module operation $A \otimes_\pi E \to E$ and a right A-module operation $E \otimes_\pi A \to E$ such that $a(xb) = (ax)b$ for $a, b \in A, x \in E$.

2. Flatness of Fréchet modules

Definition 2.1. If E and F are left A-modules, an A-module homomorphism $f: E \to F$ will be an element of $L(E, F)$ the linear space of all continuous linear maps from E to F, such that $f(ax) = af(x)$ for all $a \in A, x \in E$.

Definition 2.2. An exact sequence of A-modules will be a sequence

$$\cdots \longrightarrow E_{i-1} \stackrel{\alpha_{i-1}}{\longrightarrow} E_i \stackrel{\alpha_i}{\longrightarrow} E_{i+1} \stackrel{\alpha_{i+1}}{\longrightarrow} \cdots$$

with each E_i an A-module, each α_i a module homomorphism, and such that $ker\ \alpha_i = \text{im}\ \alpha_{i+1}$ for each i and we denote it by $(E_{\bullet}, \alpha_{\bullet})$ or simply by $E.$ if we know morphisms. If the above exact sequence is a complex ($\alpha_i \circ \alpha_{i-1} = 0$ for all i), then it split if there is a sequence of module homomorphisms $\beta_i: E_i \to E_{i-1}$, such that $\beta_{i+1} \circ \alpha_i + \alpha_{i-1} \circ \beta_i = id : E_i \to E_i$ for each i. The homology modules or homology spaces of a complex $(E., \alpha.)$ of A-modules are the A-modules

$$H_n(E.) := \frac{\ker\ \alpha_n}{im\alpha_{n+1}},\ n \in \mathbb{Z}$$

Definition 2.3. An exact sequence of A-modules will be called $\mathbb{C}$-split if it is split when considered as sequence of $\mathbb{C}$-modules.

Definition 2.4. If E is an l.c.s., then $A \otimes_\pi E$ may be considered a left A-module, where the map $a \otimes b \otimes x \to ab \otimes x: A \otimes_\pi A \otimes_\pi E \to A \otimes_\pi E$ determines the module operation. A module of this form will be called a free left A-module.

Definition 2.5. Let E be an A-module (left, right, or bi). We shall say E is projective if whenever $\alpha: F \to G$ is a surjective module homomorphism, with $0 \to ker\alpha \to F \to G \to 0$ C-split, each module homomorphism $\beta: E \to G$ lifts to a homomorphism $\gamma: E \to F$ with $\alpha \circ \gamma = \beta$.

Definition 2.6. Let E be a right A-module and F a left A-module. Let $\delta: E \otimes_\pi A \otimes_\pi F \to E \otimes_\pi F$ be defined by $\delta(x \otimes a \otimes y) = xa \otimes y - x \otimes uy$. Then we define the A-module tensor product $E \otimes_A F$ to be the vector space $\frac{E \otimes_\pi F}{im\delta}$.

If $F \stackrel{\alpha}{\longrightarrow} G$ is a left A-module homomorphism then there are induced maps $1 \otimes \alpha : E \otimes_\pi F \to E \otimes_\pi G$ and $1 \otimes 1 \otimes \alpha: E \otimes_\pi A \otimes_\pi F \to E \otimes_\pi A \otimes_\pi G$ for any right A-module E. The diagram

$$\begin{array}{ccc} E \otimes_\pi A \otimes_\pi F & \stackrel{1\otimes 1 \otimes \alpha}{\longrightarrow} & E \otimes_\pi A \otimes_\pi G \\ {\scriptstyle 1\otimes\delta}\downarrow & & \downarrow{\scriptstyle \delta} \\ E \otimes_\pi F & \stackrel{1\otimes\delta}{\longrightarrow} & E \otimes_\pi G \end{array}$$

is commutative. It follows that α induces a linear map $E \otimes_\pi F \to E \otimes_\pi G$ which we shall also call $1 \otimes \alpha$

Definition 2.7. If E is an A-module (left, right, or bi), then by a projective (free) resolution of E we shall mean a sequence of module homomorphisms

$$0 \longleftarrow E \longleftarrow X_0 \longleftarrow X_1 \longleftarrow \cdots \longleftarrow X_p \longleftarrow X_{p+1} \cdots$$

which is exact and has each X_p, projective (free). If above sequence is $\mathbb{C}$-split, we shall say it is a $\mathbb{C}$-split projective (free) resolution. A resolution (1) will often be denoted simply by $E \longleftarrow X_\bullet$

Definition 2.8. A morphism $\mathbf{f}: (X_\bullet, d_\bullet) \to (Y_\bullet, d'_\bullet)$ of complexes of A-modules is a family of A-module morphisms $f_n: X_n \to Y_n$ with the property that $d'_n \circ f_n = f_{n-1} \circ d_n (n \in \mathbb{Z})$

Definition 2.9. Let $(X_\bullet, d_\bullet)$ and $(Y_\bullet, d'_\bullet)$ be complex of unital modules, and let , $\mathbf{g} : X. \to Y.$ be two morphism between them. Then A family of morphisms of A-modules $s = \{s_n: X_n \to Y_{n+1, n\in\mathbb{Z}}\}$ is called a homotopy between $\mathbf{f}$ and $\mathbf{g}$ if $d'_n s_n + d_{n-1} s_{n-1} = f_n - g_n$ for any n. Two morphisms between complexes are said to be homotopic if a homotopy exists between them.

A morphism between complexes $\mathbf{f}: X_\bullet \to X_\bullet$ is called a homotopy equivalence if there exists $\mathbf{g}: Y_\bullet \to X_\bullet$ such that $\mathbf{f} \circ \mathbf{g}$ and $\mathbf{g} \circ \mathbf{f}$ are homoptic to the identities in $X.$ and $Y.$ respectively. The complexes $X.$ and $Y.$ are said to be homotopically equivalent if there exists a homotopy equivalence between them.

Proposition 2.10. [2, corollary III.1.4] If $X_\bullet$ and $Y_\bullet$ are homotopically equivalent, then there exists a topological isomorphism between $H_n(X_\bullet)$ and $H_n(Y_\bullet)$

Proposition 2.11. [1, Proposition 3.1.5] Every Fréchet $\mathcal{A}$-module E has a topologically free resolution which is split over $\mathbb{C}$. Any two such resolutions are homotopically equivalent.

Definition 2.12. Let $\mathcal{A}$ be a Fréchet algebra with unit, and let E, F be Fréchet $\mathcal{A}$-modules. The Bar complex $B_\bullet^{\mathcal{A}}(E, F)$ consists of the spaces

$$B_0^{\mathcal{A}}(E,F) := E \hat{\otimes}_\pi F$$
$$B_n^{\mathcal{A}}(E,F) := E \hat{\otimes}_\pi \underbrace{\mathcal{A} \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi \mathcal{A}}_{n \text{ times}} \hat{\otimes}_\pi F \; n \geq 1$$

and the boundary operators $d_n: B_n^{\mathcal{A}}(E,F) \to B_{n-1}^{\mathcal{A}}(E,F),\ n \geq 1$ defined by

$$d_n(e \otimes a_1 \otimes \cdots \otimes a_n \otimes f) := \quad a_1 e \otimes \cdots a_n \otimes f + (-1)^n e \otimes a_1 \otimes \cdots \otimes a_n f + \sum_{j=1}^{n-1} (-1)^j e \otimes \cdots \otimes a_j a_{j+1} \otimes \cdots \otimes f.$$

Lemma 2.13. Let $\mathcal{A}$ be a Fréchet algebra with unit and let E be a Fréchet A-module. The complex $B_\bullet^{\mathcal{A}}(E, \mathcal{A})$ is a topologically free resolution of E which is split over $\mathbb{C}$.

Proof. The $\mathbb{C}$-linear maps $h_n(e \otimes a_0 \otimes a_1 \cdots \otimes a_n) = 1 \otimes e \otimes a_0 \otimes a_1 \cdots \otimes a_n$ from $B_n^{\mathcal{A}}(E, \mathcal{A})$ into $B_{n+1}^{\mathcal{A}}(E, \mathcal{A})$, are continuous, and satisfy the splitting conditions $d_{n+1} h_n + h_{n-1} d_n = id_n, n \in \mathbb{N}$, where id_n denotes the identity operator on $B_n^{\mathcal{A}}(E, \mathcal{A})$ and h_{-1}, d_0 are by definition equal to zero.

Definition 2.14. Let $\mathcal{A}$ be a commutative Fréchet algebra with identity. With each pair of Fréchet $\mathcal{A}$-modules E and F one associates the locally convex spaces

$$\widehat{\mathrm{Tor}}_n^{\mathcal{A}}(E,F) := H_n\big(B_\bullet^{\mathcal{A}}(E,F)\big),\ n \in \mathbb{Z}_+$$

where H_n is n'th homology and the spaces equipped with (not necessarily Hausdorff) quotient topology.

Lemma 2.15. Let $\mathcal{A}$ be a Fréchet algebra with identity. For any Fréchet $\mathcal{A}$-module E and Fréchet space V, the following isomorphism of Fréchet $\mathcal{A}$-modules holds:

$$V \hat{\otimes}_{\mathcal{A}} \big(\mathcal{A} \hat{\otimes}_\pi E\big) \cong V \hat{\otimes}_\pi E$$

Proof. The sequence

$$\big(V \hat{\otimes}_\pi \mathcal{A}\big) \hat{\otimes}_\pi \mathcal{A} \hat{\otimes}_\pi E \xrightarrow{d_1} \big(\mathcal{A} \hat{\otimes}_\pi V\big) \hat{\otimes}_\pi E \xrightarrow{p} V \hat{\otimes}_\pi E \longrightarrow 0$$

where $p(v \otimes a \otimes e) = v \otimes ae$, is exact and $\mathbb{C}$-split.

Proposition 2.16. Let $\mathcal{A}$ be a commutative unital Fréchet algebra, and let $L_\bullet \longrightarrow E \longrightarrow 0$ be a $\mathbb{C}$-split, topologically free resolution of the Fréchet $\mathcal{A}$-module E. For every Fréchet $\mathcal{A}$-module F, we have isomorphisms of locally convex spaces

$$\mathrm{Tor}_n^{\mathcal{A}}(E,F) \cong H_n\big(L_\bullet \hat{\otimes}_{\mathcal{A}} F\big),\ n \in \mathbb{N}$$

Proof. By lemma 2.15 we have $B_\bullet^{\mathcal{A}}(E,F) \cong B_\bullet^{\mathcal{A}}(E,\mathcal{A}) \hat{\otimes}_{\mathcal{A}} F$ and by proposition 2.13 and 2.11 the complexes $B_\bullet^{\mathcal{A}}(E,\mathcal{A})$ and $L_\bullet$ are homotopically equivalent in the category of Fréchet $\mathcal{A}$-modules. It follows that the complexes $B_\bullet^{\mathcal{A}}(E,F)$ and $L_\bullet \hat{\otimes}_{\mathcal{A}} F$ are homotopically equivalent, but this time only in the category of locally convex spaces, so by proposition 2.10 the result follows.

Definition 2.17. A left A-module Y is said to be flat if, for any $\mathbb{C}$-split short exact sequence $0 \longrightarrow E_1 \longrightarrow E_2 \longrightarrow E_3 \longrightarrow 0$ of right A-modules, the sequence $0 \to E_1 \hat{\otimes}_{\mathcal{A}} F \to E_2 \hat{\otimes}_{\mathcal{A}} F \to E_3 \hat{\otimes}_{\mathcal{A}} F \to 0$ is exact.

Theorem 2.18. [2, Proposition VII.1.2] Let Y be a Fréchet $\mathcal{A}$-module then the following are equivalent:

(1) Y is flat.

(2) for every $\mathbb{C}$-split complex $X_\bullet$ of right A-modules $X_\bullet \hat{\otimes}_{\mathcal{A}} Y$ is exact.

(3) for every Fréchet right $\mathcal{A}$-module X, $\widehat{\mathrm{Tor}}_n^{\mathcal{A}}(X,Y) = 0, n \in \mathbb{N}$ and $\widehat{\mathrm{Tor}}_0^{\mathcal{A}}(X,Y)$ is Hausdorff.

Definition 2.19. Let $(I,<)$ be a partially ordered countable set, and let $(E_\alpha)_{\alpha\in I}$ be an inverse systetm of Fréchet spaces, that is, a family of Fréchet spaces $E_\alpha (\alpha \in I)$ together with a system of continuous linear maps

$$i_\alpha^\beta : E_\alpha \to E_\beta \ \alpha \ge \beta$$

such that

$$i_\alpha^\alpha = id,\ i_\beta^\gamma \circ i_\alpha^\gamma = i_\alpha^\beta,\ \alpha \geq \beta \geq \gamma$$

then the projective limit $E = \lim\limits_{\leftarrow}(E_\alpha)$ is, by definition, the closed subspace of the topological product $\Pi_{\alpha\in I} E_\alpha$ consisting of all families $(X_\alpha)_{\alpha\in I}$ with the property that $i_\alpha^\beta(x_\alpha) = x_\beta$ for all $\alpha \geq \beta$. Equipped with the relative topology of the product space, E is a Fréchet space. Let $\pi_\alpha: E \to E_\alpha$ be the canonical projection for each α. Then by definition $i_\alpha^\beta \pi_\alpha = \pi_\beta$ for all $\alpha \geq \beta$. A projective limit is called reduced if $\pi_\alpha^- E = E_\alpha$ with respect to the topology E_α.

Definition 2.20. By a morphism $u: E \to F$ of countable inverse systems $E = (E_\alpha)_{\alpha\geq 1}$ and $F = (F_\alpha)_{\alpha\geq 1}$ of Fréchet spaces we mean a family $u = (u_\alpha)_{\alpha\geq 1}$ of continuous linear maps $u_\alpha: E_\alpha \to F_\alpha$ that commute with the structural maps of E and F in the obvious sense. A morphism $u = (u_\alpha)$ is called a monomorphism if all the components u_α are topological monomorphisms.

Theorem 2.21. [1, Proposition 3.2.4] Consider a short exact sequence

$$0 \longrightarrow (E_\alpha) \stackrel{u}{\longrightarrow} (F_\alpha) \stackrel{v}{\longrightarrow} (G_\alpha) \longrightarrow 0$$

of countable inverse systems of Fréchet spaces. If all structural maps $i_\alpha^\beta: E_\alpha \to E_\beta$, $(\alpha \geq \beta \geq 1)$ have dense range, then the induced sequence

$$0 \longrightarrow \lim_{\leftarrow}(E_\alpha) \longrightarrow \lim_{\leftarrow}(F_\alpha) \longrightarrow \lim_{\leftarrow}(G_\alpha) \longrightarrow 0$$

of inverse limits remains exact.

Theorem 2.22. [1, Proposition 3.2.11] Let $\mathcal{A}$ be a commutative Fréchet algebra with identity, and let $(E_\alpha)_{\alpha\in\mathbb{N}}$ be a reduced inverse system of Fréchet $\mathcal{A}$-modules. Suppose that F is a Fréchet $\mathcal{A}$-module such that $\hat{\mathrm{Tor}}_n^{\mathcal{A}}(E_\alpha, F) = 0, n \in \mathbb{N}$ then $\hat{\mathrm{Tor}}_n^{\mathcal{A}}\left(\lim\limits_{\leftarrow}(E_\alpha), F\right) = 0$ for all $n \in \mathbb{N}$.

Corollary 2.23. Let $\mathcal{A}$ be a commutative Fréchet algebra with identity, and let $(E_\alpha)_{\alpha\in\mathbb{N}}$ be a reduced inverse system of flat Fréchet $\mathcal{A}$-modules. Then $\lim\limits_{\leftarrow}(E_\alpha)$ is flat.

Proof. For Fréchet $\mathcal{A}$-module F Let $L.$ be its $\mathbb{C}$-split free resolution since E_α is flat so by $2.18 E_\alpha \hat{\otimes}_{\mathcal{A}} L_{\bullet} \to E_\alpha \hat{\otimes}_{\mathcal{A}} F \to 0$ is exact hence by 2.21 whereas $(E_\alpha)_{\alpha\in\mathbb{N}}$ is reduced so

$$\lim_{\leftarrow} E_\alpha \hat{\otimes}_{\mathcal{A}} L_{\bullet} \to \lim_{\leftarrow} E_\alpha \hat{\otimes}_{\mathcal{A}} F \to 0$$

is exact. In the other hand since $L_i, i \geq 0$ are free so are of the form $L_i = \mathcal{A} \hat{\otimes}_\pi H_i$ for some Fréchet space H_i, hence by 2.15 and this fact that projective limit commute by projective tensor product [3, §41.6] we have:

$$\left(\lim_{\leftarrow} E_\alpha\right) \hat{\otimes}_{\mathcal{A}} L_i \cong \left(\lim_{\leftarrow} E_\alpha\right) \hat{\otimes}_{\mathcal{A}} \mathcal{A} \hat{\otimes}_{\pi} H_i \cong \left(\lim_{\leftarrow} E_\alpha\right) \hat{\otimes}_{\pi} H_i$$
$$\cong \lim_{\leftarrow}\left(E_\alpha \hat{\otimes}_{\pi} H_i\right) \cong \lim_{\leftarrow}\left(E_\alpha \hat{\otimes}_{\mathcal{A}} L_i\right)$$

so by 2 we have:

$$\left(\lim_{\leftarrow} E_\alpha\right) \hat{\otimes}_{\mathcal{A}} L_\bullet \to \lim_{\leftarrow}\left(E_\alpha \hat{\otimes}_{\mathcal{A}} F\right) \to 0$$

hence by 2.16 $\left(\lim_{\leftarrow} E_\alpha\right) \hat{\otimes}_{\mathcal{A}} F \cong \lim_{\leftarrow}(E_\alpha) \hat{\otimes}_{\mathcal{A}} F\Big)$ so $\left(\lim_{\leftarrow} E_\alpha\right) \hat{\otimes}_{\mathcal{A}} F$ is Hausdorff therefore by above theorem and part (3) theorem 2.18 $\lim_{\leftarrow} E_\alpha$ is flat.

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Effect of Energetic Electrons on Dynamic Motion of Ion-Acoustic Waves in Presence of External Periodic Force

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Abstract

The dynamic motions of ion–acoustic (IA) waves in a plasma is studied by considering the Kappa-Cairns electron distribution. The effect of external periodic force is also considered, here. Using the reductive perturbation technique, a Korteweg-de Vries (K-dV) equation is obtained. The quasiperiodic motions of IA waves are investigated by considering two-dimensional phase portraits and time-series analysis. The effects of non–thermal parameters (i.e., κ, α) and the strength (f_0) of the external periodic force on nonlinear traveling wave structures are also discussed.

Keywords

Ion-acoustic waves, dynamic motions, external periodic force, energetic electrons.

1. Introduction

Usually, in plasmas far from thermal equilibrium, the velocity distribution function of the charged particles is commonly assumed as non-Maxwellian. The existence of a population of highly energetic electrons characterized in such plasmas. Generally, the Kappa distribution [1] and the Cairns distribution [2] are two widely used non-Maxwellian models for energetic electrons. These distributions are a generalization of the Maxwellian (thermal) distribution and in certain limits, they will reduce to a Maxwellian distribution. However, there are many space plasma environments that the Kappa or the Cairns distribution functions cannot be used for the investigation of nonlinear waves. In these cases, the Combined Kappa-Cairns distribution is a more generalized distribution which may be relevant to solve this problem. Many authors have studied the properties of the combined Kappa-Cairns velocity distribution function [3-5]. They have investigated the structure of ion-acoustic solitary waves in a plasma with the Kappa-Cairns distributed electrons. However, the dynamic motion of ion-acoustic waves (IAWs) in a plasma with the Kappa-Cairns distributed electrons has not been studied so far.

There is special attention for the study of nonlinear traveling waves in plasmas and in this work, we will focus on the traveling wave. It is noted that the traveling wave properties can be affected by damping and external periodic perturbations.

Many authors investigated the dynamic behaviors of IA waves in different plasma systems using the bifurcation theory [6-7]. For example, Saha et al., [8] studied the dynamic behavior of IA waves in (e-p-i) magneto-plasmas with Kappa distributed electrons and positrons. Selim et al., [9] were considered characteristics of nonlinear IA traveling waves in a multi–component system consisting of positive and negative ions and superthermal electrons. In addition, bifurcation and quasiperiodic behaviors of IA waves in a magnetized electron–ion plasma with the combined Cairns–Tsallis distribution electrons were investigated in Ref. [10]. To the best of our knowledge, there is no work in literature to study the dynamic behaviors of IA waves in plasmas with non–thermal electrons featuring Kappa–Cairns distribution in the presence of the external periodic force. Therefore, in this study, we investigate the nonlinear properties of IA waves (through a perturbative approach) in a two-component plasma with the Kappa–Cairns distributed electrons.

The manuscript is organized as follows. The dynamical equations and the derivation of the K-dV equation are presented in Sec. 2. Control of dynamic motions is demonstrated in Sec. 3. Finally, a brief conclusion of our numerical results is given in Sec. 4.

2. Basic equations and derivation of the k-dv equation

We consider a collisionless plasma consists of cold ions and energetic electrons with the Kappa–Cairns' distribution function. The normalized basic set of equations for the dynamics of the ion particles are given as

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nu)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e-n. \tag{3}$$

The normalized kappa-Cairns distributed electron number density n_e is given by the relation [3-5]

$$n_e=(1-\Gamma_1\phi+\Gamma_2\phi^2)\left(1-\frac{\phi}{\kappa-3/2}\right)^{-\kappa+\frac{1}{2}}, \tag{4}$$

where

$$\Gamma_1=\Gamma\left(1+\frac{3}{2(\kappa-5/2)}\right), \tag{5}$$

$$\Gamma_2=\Gamma\left(1+\frac{1}{(\kappa-3/2)}+\frac{3}{4(\kappa-3/2)(\kappa-5/2)}\right), \tag{6}$$

$$\Gamma=\frac{4\alpha}{1+3\alpha\frac{(\kappa-3/2)}{(\kappa-5/2)}}. \tag{7}$$

It should be noted that the Kappa–Cairns distribution function reduces to the well–known Kappa distribution function [1] for $\alpha=0$, the Cairns non–thermal velocity distribution [2] for $\kappa\to\infty$, and the Maxwellian distribution function for $\alpha=0$ and $\kappa\to\infty$. On the other hand, it is found that this new distribution function must satisfy the conditions $\kappa>3/2$ and $\alpha<1$. In the above set of dynamic equations, $n(n_e)$ is normalized to n_0, ion fluid velocity u normalized to ion–acoustic speed $C_0=(k_BT_e/m_i)^{1/2}$, electrostatic potential ϕ is normalized by $(k_B T_e/e)$. Space and time variables are normalized by the Debye length $\lambda_D = \sqrt{k_B T_e/4\pi e^2 n_0}$ and the ion plasma period $\omega_{pi} = \sqrt{4\pi e^2 n_0/m_i}$, respectively.

To study the ion–acoustic nonlinear waves with small but finite amplitude and derive the K-dV equation, we apply the reductive perturbation technique (RPT) [11]. For this aim, the independent variables are stretched in the following form:

$$\xi = \varepsilon^{\frac{1}{2}}(x - \lambda t) \quad \text{and} \quad \tau = \varepsilon^{3/2} t\ , \tag{8}$$

where λ (normalized to C_0) is the phase velocity of the ion–acoustic waves and $\varepsilon \ll 1$ is a small parameter measuring the strength of nonlinearity. The dependent plasma variables n, u and ϕ, are expanded about their equilibrium values as power series of ε as

$$n = 1 + \varepsilon\, n^{(1)} + \varepsilon^2 n^{(2)} + \cdots, \tag{9}$$

$$u = 0 + \varepsilon\, u^{(1)} + \varepsilon^2 u^{(2)} + \cdots, \tag{10}$$

$$\phi = 0 + \varepsilon\, \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \cdots, \tag{11}$$

Substituting Eqs. (8) – (11) into basic equations (1)-(4) and equating the coefficients of similar powers of ε, one may obtain the lowest order of ε as

$$n^{(1)} = A_1 \phi^{(1)}, \qquad u^{(1)} = \lambda A_1 \phi^{(1)}, \tag{12}$$

with $A_1 = \left(\frac{\kappa - 1/2}{\kappa - 3/2} - \beta_1\right)$. We obtain the phase velocity as

$$\lambda = \sqrt{\frac{1}{\left(\frac{\kappa - 1/2}{\kappa - 3/2} - \beta_1\right)}}\ . \tag{13}$$

For the next order of ε, by eliminating the second-order perturbed quantities ($n^{(2)}, u^{(2)}$and $\phi^{(2)}$), and with the help of Eq. (12), we finally get an evolution equation for IAWs as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \mathrm{A}\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + \mathrm{B}\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0\ \ , \tag{14}$$

where the nonlinear and dispersion coefficients, respectively, are defined as

$$\mathrm{A} = \frac{3\lambda A_1}{2} - \frac{A_2}{\lambda A_1^2}\ , \tag{15}$$

$$\mathrm{B} = \frac{1}{2\lambda A_1^2}, \tag{16}$$

where $A_2 = \left\{\frac{(\kappa - 1/2)(\kappa + 1/2)}{2(\kappa - 3/2)^2} + \Gamma_2 - \Gamma_1 \left(\frac{\kappa - 1/2}{\kappa - 3/2}\right)\right\}$.

3. control of dynamic motions

There are standard techniques to study the control of dynamic motion for IAWs with the effect of an external periodic force, namely, sensitivity analysis, time-series analysis and phase portrait analysis. Here the K-dV equation (14) is reduced a dynamical system employing the alteration $\chi = \xi - \mathrm{U}\tau$. Substituting the latter transformation in the equation (14), we get the following expression

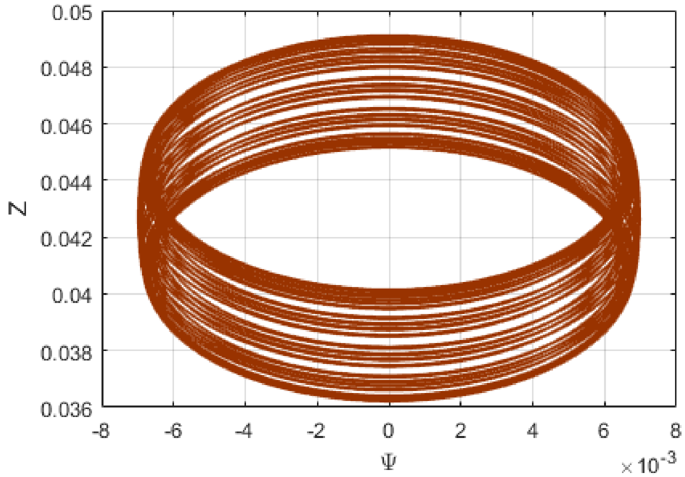
$$-\mathrm{U}\frac{d\phi^{(1)}}{d\chi} + \mathrm{A}\phi^{(1)}\frac{d\phi^{(1)}}{d\chi} + \mathrm{B}\frac{d^3\phi^{(1)}}{\partial\chi^3} = 0\,. \qquad (17)$$

Now, introducing an external periodic force $f_0\cos(\omega\chi)$ in the system (17), the following perturbed dynamical structure is obtained as

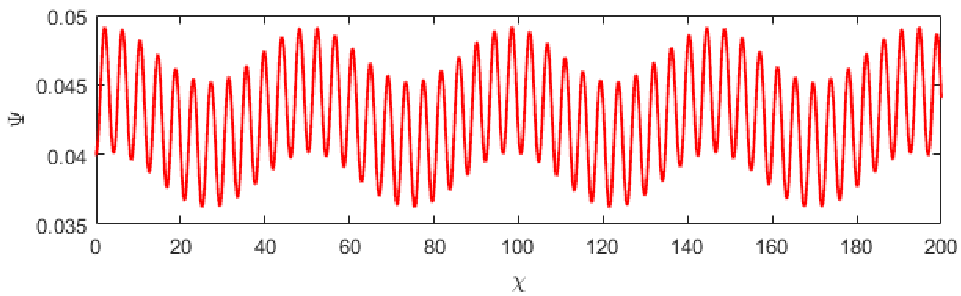
$$\begin{cases} \dfrac{d\psi}{d\chi} = Z\,, \\ \dfrac{dZ}{d\chi} = \dfrac{1}{\mathrm{B}}\left(\mathrm{U}\psi - \dfrac{1}{2}\mathrm{A}\psi^2\right) + f_0\cos(\omega\chi), \end{cases} \qquad (18)$$

where $\psi(\chi) = \phi^{(1)}$. Here, ω denotes frequency and f_0 denotes intensity of the external periodic force implemented in the system. We should note that the system Eq. (18) is a planar dynamical system and the phase orbits defined by the vector fields of Eq. (18) will determine all traveling wave solutions of Eq. (14). We study the bifurcations of phase portraits of (18) in the $(\phi^{(1)}, Z)$ phase plane when the parameters A, B, and U are clear for us. The coefficients A and B are functions from the parameters κ and α, and hence, it is difficult to explore the system for a complete range of parametric space. Therefore, we investigate our numerical analysis for some fixed values of these parameters. For investigation of the control of dynamic motions of the structure (18), different techniques, such as (i) phase portrait and (ii) time-series analysis can be used [6, 12]. A geometric structure of the trajectories of a three-dimensional dynamical system is shown by analyzing a 2D phase portrait in phase space. In the phase portrait, each set of initial conditions is represented by a different curve or point. It consists of a plot of the trajectories in the state space. This gives the information about whether there is an attractor, a repellor, or a limit cycle for a set of parameter values. On the other hand, a time-series analysis shows a series of data points indexed in time series. In fact, nonlinear time-series analysis allows to extract from the measured time series the physical properties of the system that generated them.

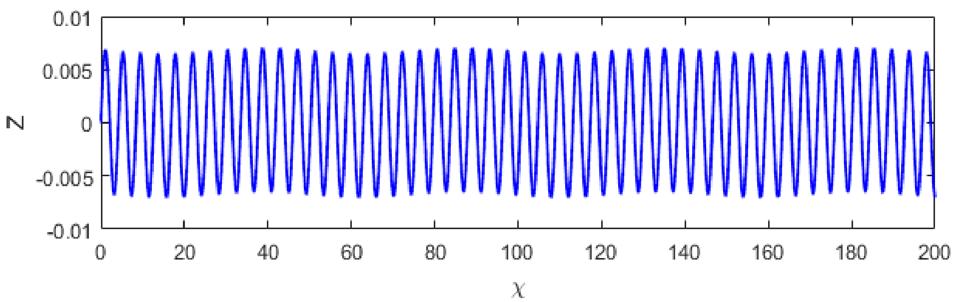
Figures 1(a)-(c), present the phase portrait, the behavior of potential Ψ, and electric field Z in a collisionless nonthermal plasma with the Kappa-Cairns distributed electrons, respectively. Figure 1(a) displays that the ion trajectories lie on the surface of the torus which is a sign of the quasiperiodic solution. In other words, this figure confirms that the system (18) exhibits a quasiperiodic behavior when an external periodic force is considered. In Figs. 1(b)- (c), time-series analysis of Ψ and Z versus χ are presented. A multi-frequency but finite oscillation is observed in Figs. 1(b)-(c).



(a)



(b)



(c)

Figure 1. 2D plot of the phase orbits and (b)-(c) time-series analysis of Ψ and Z vs. χ of the system (18) for initial conditions: $(\Psi, Z) = (0.04, 0.0001)$ with $\kappa = 5$, $U_0 = 0.01$, $\alpha = 0.076$, $f_0 = 0.01$ and $\omega = 1.5$.

In continue, we examine the influence of , α, and f_0 on the potential of nonlinear traveling waves. For this aim, we presented a time-series analysis of electric potential Ψ versus χ for different values of κ, α, f_0 in Figs 2-4, respectively.

Figure 2, represents the variation of the potential of nonlinear waves for different values of κ in the presence of an external periodic force with $f_0 = 0.01$ and $\omega = 1.5$. Here, red and blue curves are plotted for $\kappa = 5$ and $\kappa = 8$, respectively. It is observed that by increasing the spectral index κ, the amplitude of the nonlinear ion–acoustic waves will be increased.

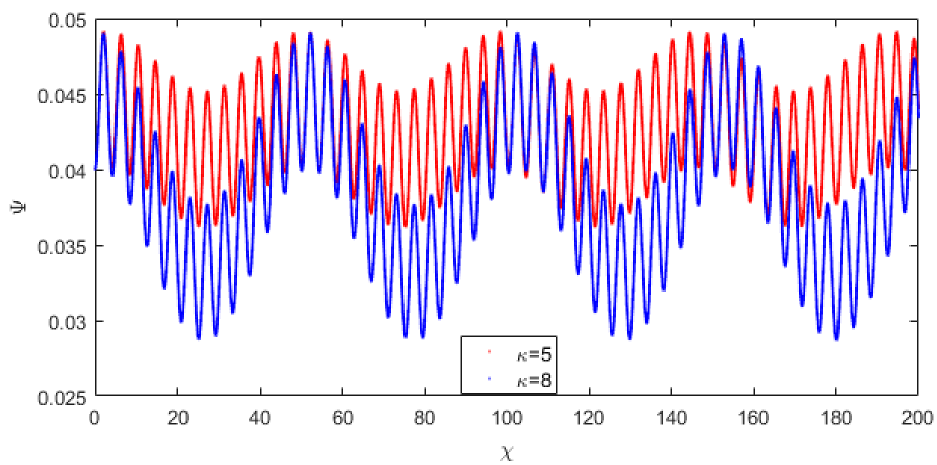


Figure 2. Profile of traveling waves potential for (a) $\kappa = 5$ (red curve) and (b) $\kappa = 8$ (blue curve). Other plasma parameters are the same as Fig. 1.

In Fig. 3, the effect of the increase in the density of the energetic electrons via parameter α in the presence of an external periodic force is studied. It is seen that the amplitude of nonlinear waves decreases as parameter α increases. Therefore, from the comparison of Figs. 2 and 3, we obtain that non–thermal Kappa-Cairns distributed electrons have significant effects on the evolution of the IA traveling waves.

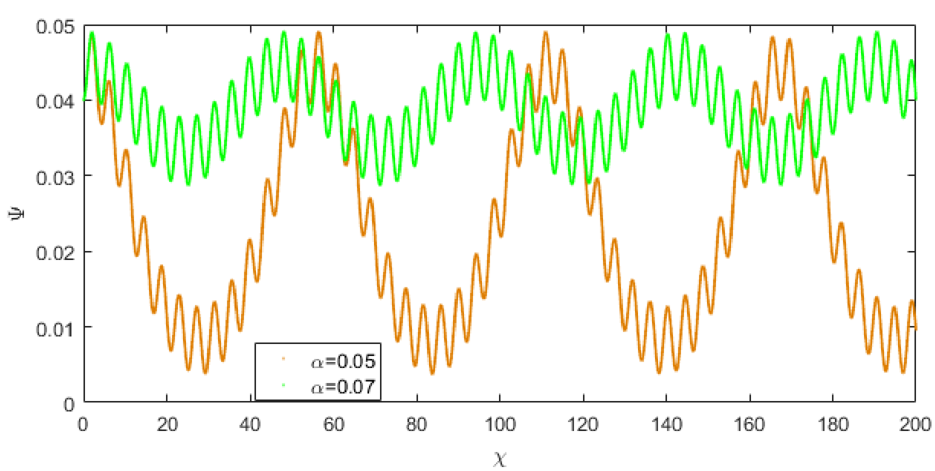


Figure 3. Profile of traveling waves potential for (a) $\alpha = 0.05$ (brown curve) and (b) $\alpha = 0.07$ (green curve). Other plasma parameters are the same as Fig. 1.

Now, the effect of the strength of external periodic force f_0 is investigated in Fig. (4). It is found that if the strength of the external force gradually increases, the amplitude of the periodic waves will be increased. It happens because, increasing of strength of external force enhancements the IAW potential energy.

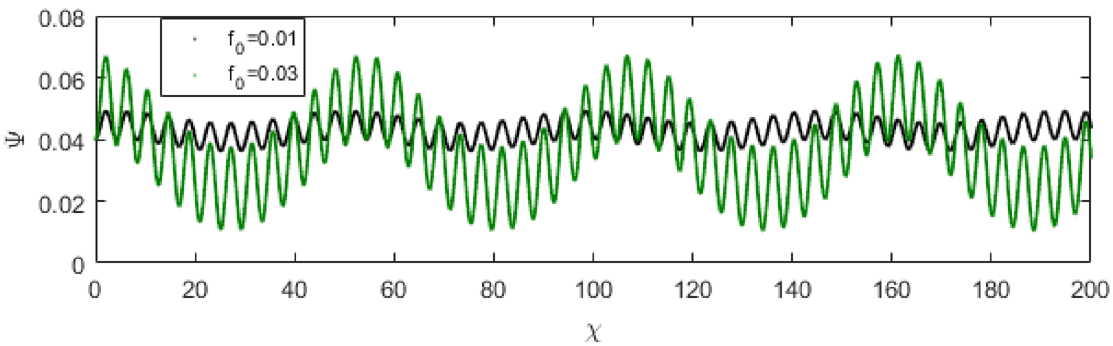


Figure 4. Variations of the potential of the nonlinear traveling waves vs. χ for different values of the amplitude of the external force (a) $f_0 = 0.01$ (black curve) and (b) $f_0 = 0.03$ (green curve). Other plasma parameters are the same as Fig. 1.

4. conclusions

The structures and dynamic motions of the nonlinear ion-acoustic waves are studied in a non-Maxwellian plasma containing cold ions and Kappa-Cairns distributed electrons, in the presence of an external periodic force. Using RPT, the K-dV equation for ion-acoustic waves is derived. The effects of non–thermal parameters (i.e., κ, α) and the amplitude f_0 of the periodic force on IA wave structures were discussed through numerical simulations. It is observed that these parameters have remarkable effects on the nonlinear structure of the IA waves in non-Maxwellian plasmas. In other words, they play a crucial role in the control of the dynamic motions of the system (18).

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New exact soliton solutions by analytical method to the Ginzburg-Landau equation

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Abstract
The Backlund transformation method of Riccati equation is an efficient method for obtaining exact solutions of some nonlinear partial differential equations. These results are going to be very useful in various areas of hydrodynamics, plasma physics, molecular biology, quantum mechanics, nonlinear optics and others.

Keywords
Soliton, Ginzburg-Landau equation, Riccati equation.

Introduction:
The objective of this article is to present new extension of the Backlund transformation method of Riccati equation to construct the exact traveling wave solutions for NLEEs in mathematical physics. The importance of our present work is, in order to generate many new and more general exact traveling wave solutions, new extension of the Backlund transformation method of Riccati equation is proposed. For illustration and to show the advantages of the proposed method, the 2D Ginzburg-Landau equation has been investigated and constructed a rich class of new traveling wave solutions [1-6].

Application to the 2D Ginzburg-Landau equation

Now we consider a class of nonlinear partial differential equation with constant coefficients which is called Ginzburg-Landau equation. As we all know Ginzburg-Landau equation is a class of a Schrodinger equation with a nonlinear term [20]

$$u_t - iu_t + \frac{1}{2}u_{xx} + \frac{1}{2}(\beta - if)u_{yy} + (1-i\delta)|u|^2 u = i\gamma u. \quad (9)$$

Suppose that Eq. (9) has an exact solution in the form

$$u = expi(\eta)v(\xi),\ \eta = (px + qy + st). \quad (10)$$

Where $v(x, y, t)$ is a real function and p, q, s are constants to be determined. Substituting (10) into Eq. (9) and canceling $expi(\eta)$, gives the partial different equation for v

$$\begin{aligned} &iv_t + \frac{1}{2}(v_{xx} + \beta v_{yy}) - \frac{1}{2}ifv_{yy} + i(pv_x + \beta v_y) + \\ &fqv_y + i(\frac{1}{2}fq^2 - \gamma)v - [s + \frac{1}{2}(p^2 + \beta q^2)]v + v^3 - i\delta v^3 = 0. \end{aligned} \quad (11)$$

Driving the Eq. (11) into real parts and imaginary parts, we have

$$\frac{1}{2}(v_{xx}+\beta v_{yy})+fqv_y+v^3-[s+\frac{1}{2}(p^2+\beta q^2)]v=0,$$
$$v_t-\frac{1}{2}fv_{yy}+(pv_x+\beta v_y)-\delta v^3+\frac{1}{2}(fq^2-\gamma)v=0. \tag{12}$$

Next we seek firstly the traveling wave solutions in the form

$$v(x,y,t)=U(\xi),\ \xi=kx+ly+vt, \tag{13}$$

Where $k,\ l,\ v,\ \xi_0$, are constants. Substituting (13) into Eqs.(12), we have the ordinary differential equations for $U(\xi)$

$$(k^2+\beta l^2)U''+fqlU'+U^3-[s+\frac{1}{2}(p^2+\beta q^2)]U=0,$$
$$fl^2U''+(pk+\beta ql+v)U'-\delta U^3+(\frac{1}{2}fq^2-\gamma)U=0. \tag{14}$$

Under the constraint conditions:

$$r=\frac{2fgl}{k^2+\beta l^2}=\frac{2(pk+\beta ql+v)}{fl^2},b=-\frac{2}{k^2+\beta l^2}=-\frac{2\delta}{fl^2},$$
$$c=\frac{s+\frac{1}{2}\left(p^2+\beta q^2\right)}{k^2+\beta l^2}=\frac{fg^2-2\gamma}{fl^2} \tag{15}$$

We can get

$$U''=rU'+bU^3+cU. \tag{16}$$

Balancing the highest order derivative U'' and U^3 nonlinear term from equation (16), we obtain $3n=n+2$, which gives $n=1$. So

$$U=A_0+A_1\left(m+\psi(\xi)\right)+B_1\left(m+\psi(\xi)\right)^{-1} \tag{17}$$

Now substituting Eq. (17) along with Eq. (8) into Eq. (16), we get a polynomial in $F(\xi)$. Equating the coefficient of same power of $\psi^i(\xi)(i=0,\pm1,\pm2,\ldots)$, we attain the system of algebraic equations, and by solving these obtained system of equations for A_0,A_1,B_1,m and σ, and by solving obtained system we get the following values:

$$A_0=r\sqrt{\frac{c}{2}},\qquad A_1=\frac{2-r}{3br}\sqrt{\frac{2}{c}}$$
$$B_1=\sqrt{\frac{2}{c}}\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)},$$
$$\sigma=\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}. \tag{18}$$

Now by using these sets of solutions for A_0, A_1, B_1, m , σ and by using the applied method we have following solutions for 2D Ginzburg-Landau equation

$$u_1 = e^{i(px+qy+st)}\left\{ r\sqrt{\frac{c}{2}} + \frac{2-r}{3br}\sqrt{\frac{2}{c}} \times \right.$$

$$\left(m + \frac{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}B + D\left(\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\tan\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)}{D + B\left(\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\tan\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)} \right) +$$

$$\sqrt{\frac{2}{c}}\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)} \times$$

$$\left.\left(m + \frac{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}B + D\left(\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\tan\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)}{D + B\left(\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\tan\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)} \right)^{-1} \right\},$$

Finally from last part of Eq. (2) we have

$$u_2 = e^{i(px+qy+st)}\left\{ r\sqrt{\frac{c}{2}} + \frac{2-r}{3br}\sqrt{\frac{2}{c}} \times \right.$$

$$\left(m + \frac{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}B + D\left(-\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\cot\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)}{D + B\left(-\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\cot\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)} \right) +$$

$$\sqrt{\frac{2}{c}}\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)} \times$$

$$\left. \left(m + \frac{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}B + D\left(-\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\cot\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)}{D + B\left(-\sqrt{\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\cot\left(\sqrt{-\frac{rc^2\left(2+3br^2\right)}{4\left(rc+r-2\right)}}\left(kx+ly+vt\right)\right)\right)} \right)^{-1} \right\},$$

Fig. 1. Periodic profile of solution u_1 of Ginzburg-Landau equation equation

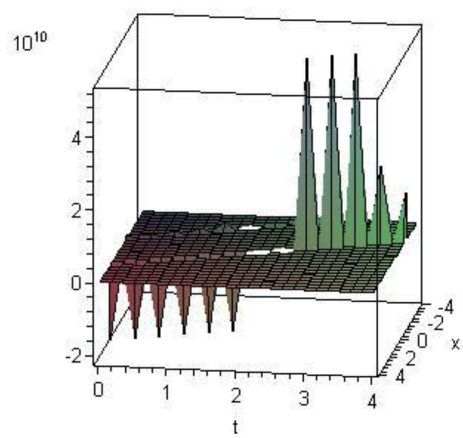
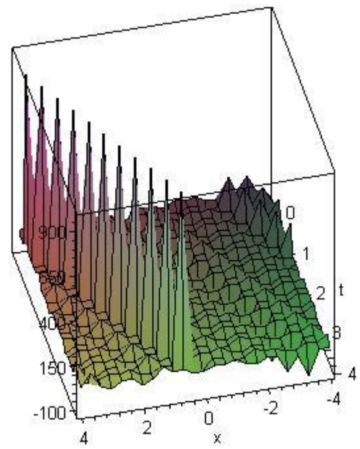


Fig.2. Soliton profile of solution u_2 of Ginzburg-Landau equation



Conclusion:

The obtained solutions for this problem are given as rational function, bright soliton and dark soliton solutions. It can be said that the results in this paper provide good supplements to the existing literature.

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An efficient method to the cubic-quintic nonlinear Schrödinger equation

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Abstract
In this paper, wave transformation has been implemented to convert nonlinear partial fractional differential equation into nonlinear ordinary differential equations. Then, the Backlund transformation method of Riccati equation has been implemented, to celebrate the soliton solutions of this system.

Keywords
Backlund transformation method; Riccati equation; nonlinear Schrödinger equation.

1. Introduction

The nonlinear Schrodinger equations (NLSEs) are important physical models, illustrate the dynamics of optical soliton promulgation in nonlinear optical fibers. In non-linear optical fibers, the optical soliton propagation is a topic of huge present curiosity since of the broad applications of ultrafast signal routing systems and short light pulses to telecommunication.
In recent years, exact solutions may help to find new phenomena. Many powerful methods for obtaining these exact solutions are presented, such as [1-10].
The rest of this paper is organized as follows: In Section 2, Backlund transformation method through with Riccati equation discussed in detail. As an application of this method, the new exact solutions of the high dispersive cubic-quintic nonlinear Schrödinger equation has been discussed in Section 3. Finally, we briefly make a conclusion to the results that have been obtained in Section 4.

2. Application of the analytical method to the high dispersive cubic-quintic nonlinear Schrödinger equation.

In this paper we discuss the solitary wave solutions for a class of high dispersive cubic-quintic nonlinear Schrodinger equations describing the ultrashort light pulse propagation as in the following

$$U_x = -i\frac{\beta_2}{2}U_{tt} + i\gamma_1|U|^2U + \frac{\beta_3}{6}U_{ttt} + i\frac{\beta_4}{24}U_{tttt} - i\gamma_2|U|^4U, \qquad \textbf{(1)}$$

where $U(x,t)$ is the slowly varying envelope of the electric field, β_2 is the parameter of the group velocity dispersion, β_3 and β_4 are, respectively, the third-order and fourth-order dispersions, γ_1 and γ_2 are the nonlinearity coefficients. When the higher order terms are ignored, we obtain the NLS. Let us now solve equation (1) by using the Backlund transformation method. To this end, we use the wave transformation

$$U(x,t)=e^{i(\omega_0 x-\omega t)}\phi(\xi), \qquad \xi=\upsilon_0 x-\upsilon t \qquad (9)$$

to reduce equation (1) to the following separating real and imaginary parts, ODE:

$$l_1\varphi'(\xi)+l_3\varphi'''(\xi)=0, \tag{10}$$

$$l_0\varphi(\xi)+l_2\varphi''(\xi)+l_4\varphi''''(\xi)+\gamma_1\varphi^3(\xi)-\gamma_2\varphi^5(\xi)=0, \tag{11}$$

Where

$$l_0=\frac{1}{2}\omega^2\left(\beta_2+\frac{1}{3}\beta_3\omega+\frac{1}{12}\beta_4\omega^2\right)-\omega_0, l_1=\omega\upsilon\left(\beta_2+\frac{1}{2}\beta_3\omega+\frac{1}{6}\beta_4\omega^2\right)-\upsilon_0,$$
$$l_2=-\upsilon^2\left(\frac{1}{2}\beta_2+\frac{1}{2}\beta_3\omega+\frac{1}{4}\beta_4\omega^2\right), l_3=-\frac{1}{6}\upsilon^3(\beta_3+\beta_4\omega), l_3=\frac{1}{24}\beta_4\upsilon^4.$$

Suppose that $l_1=0$, and $l_3=0$ so we get

$$\omega=-\frac{\beta_3}{\beta_4}, \qquad \upsilon_0=\frac{\upsilon\beta_3\left(\beta_3^{\ 2}-3\beta_2\beta_4\right)}{3\beta_4^{\ 2}},\ l_0=-\omega_0-1/8\frac{\upsilon^2\left(\beta_3^{\ 2}-2\beta_2\beta_4\right)}{\beta_4^{\ 3}}$$
$$,\ l_4=1/24\beta_4\upsilon^4\ \ l_2=\frac{\upsilon^2\left(\beta_3^{\ 2}-2\beta_2\beta_4\right)}{\beta_4^{\ 3}}$$

By considering the relations above, the real and imaginary parts (10) and (11) become

$$l_0\phi(\xi)+l_2\phi''(\xi)+l_4\phi''''(\xi)+\gamma_1\varphi^3(\xi)-\gamma_2\varphi^5(\xi)=0 \tag{12}$$

Balancing the highest order derivative ϕ'''' and nonlinear term ϕ^5 in equation (12), we obtain, $N=1$. So from (6)

$$\phi(\xi)=A_0+A_1\psi(\xi) \tag{13}$$

Now substituting Eq. (13) along with Eq. (8) into Eq. (12), we get a polynomial in $\psi(\xi)$. Equating the coefficient of same power of $\psi^i(\xi)(i=0,1,2,\ldots)$, we attain the system of algebraic equations, and by solving these obtained system of equations for A_0, A_1, ω and υ, and by solving obtained system we get the following values:

$$A_0=\frac{\sqrt{20}}{10\sqrt{\beta_4\upsilon^4\gamma_2^{\ 3}}\,\gamma_2}\times$$
$$\left(\sqrt{\beta_4\upsilon^4\gamma_2^{\ 3}}\,\gamma_2\left(\left(\frac{1}{2}\gamma_1\sqrt{\beta_4\upsilon^4\gamma_2^{\ 3}}+\frac{1}{8}\frac{\upsilon^2\left(\beta_3^{\ 2}-2\beta_2\beta_4\right)}{\beta_4^{\ 3}}+\frac{5}{6}\beta_4\upsilon^4\sigma\gamma_2^{\ 2}\right)\right)^{\frac{1}{2}}\right) \tag{14a}$$

$$A_1 = -\frac{\sqrt[4]{\beta_4 \upsilon^4 \gamma_2^{\,3}}}{\gamma_2} \tag{14b}$$

$$\upsilon = \frac{1}{10}\frac{\sqrt{1550}\sqrt{\dfrac{2\gamma_2\beta_2\beta_4 - \beta_3^{\,2}\gamma_2 + 4\beta_4^{\,3}\gamma_1\sqrt{\gamma_2\beta_4}}{\sigma\gamma_2}}}{\beta_4^2} \tag{14c}$$

Now by substituting the Eqs. (14a-14b-14c) into (13) along with general solutions of Riccati equation in (2b) we have solutions of Eq. (1) as follows

$$\phi(\xi) = \frac{\sqrt{20}}{10\sqrt{\beta_4\upsilon^4\gamma_2^{\,3}}\,\gamma_2}\left(\sqrt{\beta_4\upsilon^4\gamma_2^{\,3}}\,\gamma_2\left(\left(\frac{1}{2}\gamma_1\sqrt{\beta_4\upsilon^4\gamma_2^{\,3}} + \frac{1}{8}\frac{\upsilon^2\left(\beta_3^{\,2} - 2\beta_2\beta_4\right)}{\beta_4^{\,3}} + \frac{5}{6}\beta_4\upsilon^4\sigma\gamma_2^{\,2}\right)\right)^{\frac{1}{2}}\right) +$$
$$-\frac{\sqrt[4]{\beta_4\upsilon^4\gamma_2^{\,3}}}{\gamma_2}\frac{-\sigma B + D\varphi(\xi)}{D + B\ \varphi(\xi)}$$

From method applied for $\sigma = 0$, we have

$$U_1(x,t) = \frac{\sqrt{20}}{10\sqrt{\beta_4\upsilon^4\gamma_2^{\,3}}\,\gamma_2} e^{i(\omega_0 x - \omega t)} \times$$
$$\left(\sqrt{\beta_4\upsilon^4\gamma_2^{\,3}}\,\gamma_2\left(\left(\frac{1}{2}\gamma_1\sqrt{\beta_4\upsilon^4\gamma_2^{\,3}} + \frac{1}{8}\frac{\upsilon^2\left(\beta_3^{\,2} - 2\beta_2\beta_4\right)}{\beta_4^{\,3}} + \frac{5}{6}\beta_4\upsilon^4\sigma\gamma_2^{\,2}\right)\right)^{\frac{1}{2}}\right) +$$
$$-\frac{\sqrt[4]{\beta_4\upsilon^4\gamma_2^{\,3}}}{\gamma_2}\frac{-\sigma B - D\dfrac{1}{(\upsilon_0 x - \upsilon t) + \bar{\omega}}}{D - B\dfrac{1}{(\upsilon_0 x - \upsilon t) + \bar{\omega}}} e^{i(\omega_0 x - \omega t)}$$

For $\sigma > 0$

$$U_2(x,t)=\frac{\sqrt{20}}{10\sqrt{\beta_4\upsilon^4\gamma_2^{\ 3}}\ \gamma_2}e^{i(\omega_0 x-\omega t)}\times$$

$$\left(\sqrt{\beta_4\upsilon^4\gamma_2^{\ 3}}\ \gamma_2\left(\left(\frac{1}{2}\gamma_1\sqrt{\beta_4\upsilon^4\gamma_2^{\ 3}}+\frac{1}{8}\frac{\upsilon^2\left(\beta_3^{\ 2}-2\beta_2\beta_4\right)}{\beta_4^{\ 3}}+\frac{5}{6}\beta_4\upsilon^4\sigma\gamma_2^{\ 2}\right)\right)^{\frac{1}{2}}\right)+$$

$$-\frac{\sqrt[4]{\beta_4\upsilon^4\gamma_2^{\ 3}}}{\gamma_2}\frac{-\sigma B+D\sqrt{\sigma}\tan\left(\sqrt{-\sigma}\left(\upsilon_0 x-\upsilon t\right)\right)}{D+B\sqrt{\sigma}\tan\left(\sqrt{-\sigma}\left(\upsilon_0 x-\upsilon t\right)\right)}e^{i(\omega_0 x-\omega t)}$$

Conclusions:

This study has been shown that the applied methods are effective, simple and more wide-ranging than the Backlund transformation method because it yields many new solutions. Therefore, we conclude that the Backlund transformation method can be applied to study many other NFDEs.

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Exact soliton solutions of extended K-dV equation by Riccati equation method

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Abstract

We introduce the extended K-dV equation with help of the reductive perturbation method, which admits a double layer structure in current plasma model. The obtained solutions are expressed by the hyperbolic, trigonometric and rational functions. Our calculations show that, considered method is straightforward to solve nonlinear partial differential equations

Keywords

Exact solution, Korteweg-de Vries(K-dV) equation, Riccati equation method.

1. Introduction

Exact traveling waves solution methods of nonlinear PDEs have become more and more important resulting in methods like the extended tanh-function method [1-2], variational iteration method [3–5], Kudryashov method [6-8], Sub-equation method [9], Auxiliary equation method [10], first integral method [11]. These methods have become a necessary part of applied mathematics and mathematical physics.

The main idea of this paper is finding the exact solutions for double layers structure model of extended Kd-V equation by using of the new extension of the Riccati equation method see [21-26].

In Section 2, we first set up the basic structure for derivation the extended K-dV equation as double layers.

In Section 3, we explain the structure of the considered method and application to the obtained extended Kd-V equation in plasma systems. Finally, we make some general concluding remarks in Sec. 5.

2. Basic structure of extended K-dV equation

According to the structure of this paper we consider the plasma consisting of cold ions and two distinct group of electrons, cold electrons (n_c, T_c) and hot electrons (n_h, T_h), here the Lorentzian (kappa) distribution assumed for electrons is:

$$\frac{\partial \varphi_1}{\partial \tau} + P\,\frac{\partial \varphi_1^2}{\partial \xi} + Q\,\frac{\partial \varphi_1^3}{\partial \xi} + R\,\frac{\partial^3 \varphi_1}{\partial \xi^3} = 0\,, \tag{19}$$

and

$$P = \frac{\lambda^3}{2}\Big[\frac{3}{2\lambda^4} - \big(C_2(1-f) + D_2 f\,\sigma^2\big)\Big]\ ,$$

$$Q=\frac{\lambda^3}{2}\left(\frac{5}{2\lambda^6}-C_3(1-f)-f\,\sigma^3D_3\right),$$

$$R=\frac{\lambda^3}{2}, \tag{20}$$

$$C_3=\frac{(\kappa_c-\frac{1}{2})(\kappa_c+\frac{1}{2})(\kappa_c+\frac{3}{2})}{6(\kappa_c-\frac{3}{2})^3},$$

$$D_3=\frac{(\kappa_h-\frac{1}{2})(\kappa_h+\frac{1}{2})(\kappa_h+\frac{3}{2})}{6(\kappa_h-\frac{3}{2})^3}$$

3. Method application

The main steps of modified Riccati equation method combined with the algebra expansion are as follows:
Step 1. At first we consider a general nonlinear partial differential equation of the form

$$F\left(u_t,u_t,u_x,u_{tt},u_{xx},\ldots\right)=0 \tag{21}$$

By using the wave transformation

$$u(x,t)=U(\xi),\qquad \xi=x-\lambda t.$$

which can be converted to an ODE as the following form

$$G\left(u,u',u'',\ldots\right)=0, \tag{22}$$

Step 2. Suppose that the solution of Eq. (22) can be presented as follows

$$U(\xi)=\sum_{k=0}^{m}b_kF^k,\quad \text{and}\qquad F=a^{f(\xi)} \tag{23}$$

where $b_k\left(0\le k\le m\right)$ are constant coefficients to be determined later and $f(\xi)$ satisfies the following ODE

$$f'(\xi)=\frac{1}{\ln(a)}\left(\alpha a^{-f(\xi)}+\beta+\sigma a^{f(\xi)}\right). \tag{24}$$

Above ODE has a wide range of exact solutions. Here just some of them are listed.

Family 1: Trigonometric function solutions

If $\beta^2-4\alpha\sigma<0$ and $\sigma\neq 0$, then ,

$$F_9=\frac{-\beta}{2\sigma}+\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\tan\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2}\xi\right),$$

or

$$F_{10}=\frac{-\beta}{2\sigma}-\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2\sigma}\cot\left(\frac{\sqrt{-\left(\beta^2-4\alpha\sigma\right)}}{2}\xi\right).$$

If $\alpha\sigma>0,\ \sigma\neq 0$ and $\beta=0$, then

$$F_{15}=\sqrt{\frac{\alpha}{\sigma}}\tan\left(\sqrt{\alpha\sigma}\xi\right),$$

or

$$F_{16}=-\sqrt{\frac{\alpha}{\sigma}}\cot\left(\sqrt{\alpha\sigma}\xi\right).$$

Family 2: Rational function solutions:

If $\beta=0$ and $\alpha=-\sigma$, then

$$F_{17}=\frac{e^{2\alpha\xi}+1}{e^{2\alpha\xi}-1}.$$

If $\alpha=\sigma=0$, then

$$F_{18}=\frac{-(1+e^{2\beta\xi})}{2e^{\beta\xi}}.$$

Step 3. Determine the positive integer m in Eq. (23). It can be done by balancing the highest order derivative term and the highest order nonlinear term in (3).If the degree of $u(\xi)$ is $D[u(\xi)]=n$,then the degree of the other expressions will be given by

$$D\left[\frac{d^{\lambda}u(\xi)}{d\xi^{\lambda}}\right]=n+\lambda,\quad \text{and} \qquad D\left[u^{\lambda}\left(\frac{d^{\upsilon}u(\xi)}{d\xi^{\upsilon}}\right)^{s}\right]=n\lambda+s(n+\upsilon). \tag{25}$$

Therefore, we can find the value of m in equation (3), using equation (4).

Step 4. By substituting the Eq. (23) along with its required derivatives into Eq. (24) and compare the coefficients of powers of $f(\xi)$ in resultant equation for obtaining the set of algebraic equations.

Step 5. Solve the set of algebraic equations using the Maple package and put the results generated in Eq. (24) to extract the exact solutions of Eq. (21).

Now from above, we will exert the analytical method to obtain new and more general exact solutions and then the solitary wave solutions of the extended K-dV equation (19).

For this aim we by combining the variables ξ and τ into one variable $\chi = \xi - V\tau$ (V is the wave velocity) and to integrating with respect to χ, Eq. (19) is transformed as

$$-V\varphi_1 + P\varphi_1^2 + Q\varphi_1^3 + R\varphi_1'' + s = 0 \tag{26}$$

Where $\varphi_1' = d\varphi_1 / d\chi$.

Thus using (26) and considering the homogeneous balance between φ_1^3 and $d^2\varphi_1 / d\chi^2$ in Eq. (26) we obtain that $n = 1$. This, indubitably, allows us to assume that the solution is in form

$$\varphi_1(\chi) = b_0 + b_1 a^{f(\chi)}, \tag{27}$$

Now substituting Eq. (27) along with Eq. (24) into Eq. (26), we get a polynomial in $F(\chi)$. Equating the coefficient of same power of $a^{if(\xi)}(i = 0,1,2,...)$, we attain the system of algebraic equations, and by solving these obtained system of equations for $\alpha_0, \alpha_1, \beta_1,$ and m , and by solving obtained system we get the following values:

$$\begin{aligned} b_1 &= \pm\frac{\sqrt{-2QR}\,\delta}{Q}, \\ b_0 &= \pm\frac{1}{6}\frac{-2P + 3\sqrt{-2QR}\,\beta}{Q}, \end{aligned} \tag{28}$$

Trigonometric function solutions:

Family 1-2: By using **set 1** and Eq. (27) along with family 2 we have solutions of Eq. (19) as follow

If $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$, then

$$\varphi_{1,5}(\zeta,\tau) = \pm\frac{1}{6}\frac{-2P + 3\sqrt{-2QR}\,\beta}{Q} \pm \frac{\sqrt{-2QR}\,\delta}{Q}\left(\frac{-\beta}{2\sigma} + \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2\sigma}\tan\left(\frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2}(\xi - V\tau)\right)\right),$$

If $\alpha\sigma > 0, \sigma \neq 0$ and $\beta = 0$, then

$$\varphi_{1,6}(\zeta,\tau)=\pm\frac{1}{6}\frac{-2P+3\sqrt{-2QR}\,\beta}{Q}\pm$$
$$\frac{\sqrt{-2QR}\,\delta}{Q}\left(\sqrt{\frac{\alpha}{\sigma}}\tan\left(\sqrt{\alpha\sigma}\,(\xi-V\,\tau)\right)\right),$$

Rational function solutions:

Family 1-3: By using **set 1** and Eq. (27) along with family 3 we have solutions of Eq. (19) as follow

If $\beta=0$ and $\alpha=-\sigma$, then

$$\varphi_{1,10}(\zeta,\tau)=\pm\frac{1}{6}\frac{-2P+3\sqrt{-2QR}\,\beta}{Q}\pm$$
$$\frac{\sqrt{-2QR}\,\delta}{Q}\left(\frac{e^{2\alpha(\xi-V\,\tau)}+1}{e^{2\alpha(\xi-V\,\tau)}-1}\right),$$

If $\alpha=\sigma=0$, then

$$\varphi_{1,11}(\zeta,\tau)=\pm\frac{1}{6}\frac{-2P+3\sqrt{-2QR}\,\beta}{Q}\pm$$
$$\frac{\sqrt{-2QR}\,\delta}{Q}\left(\frac{-(1+e^{2\beta(\xi-V\,\tau)})}{2e^{\beta(\xi-V\,\tau)}}\right),$$

Conclusion:

In this present study we find new exact solutions for double layers structure model of extended Korteweg-de Vries(K-dV) equation. The Riccati equation method is used to find a new exact solution. So, we can say that the offered method can be extended to solve the problems of nonlinear partial evolution equations which happening in the theory of solitons and other areas.

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T-146

Exact soliton solutions to the nonliear fractional Schrödinger equation

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Abstract

In this article, Authors study the NSE, which is given in the following form

$$iD_y^{\alpha}u+\frac{1}{2}\left(D_x^{2\alpha}-D_t^{2\alpha}\right)u+|u|^2u=0,\qquad 0<\alpha\leq 1 \qquad (1)$$

It is notable that this equation encompasses a wide range of well-known equations through some specific selection of parameters. So far, a variety of techniques have been used successfully to find the exact solutions to the NS equation (1).

Keywords
Nonlinear Schrödinger equation, Sine-Gordon method.

1. Introduction

Exact solutions can be used to illustrate many nonlinear phenomena observed in mathematical physics. One of the most appropriate tools for describing many events in nature is to employ differential equations. This importance has made the traces to such equations tangible in many branches of science, including mathematics, physics [1-3], electrical engineering, astronomy, mechanics, economics, and many other existing disciplines [4]-[6]. Based on these remarkable effects, several analytical methods have been successfully applied to obtain exact solutions of such equations. Some of these methods are homotopy analysis method [7], the variational iteration method [8], the exp-function method [9], Logistic function method [10],

2. The conformable derivative

Khalil proposed an interesting definition of derivative called conformable derivative [1]. This derivative can be considered to be a natural extension of the classical derivative. Furthermore, conformable derivative satisfies all the properties of the standard calculus, for instance, the chain rule.

Definition 2.1 Let $f:[0,\infty)\to\mathbb{R}$, the conformable derivative of a function $f(t)$ of order α, is defined as

$$D_t^{\alpha}f(t)=\lim_{\epsilon\to 0}\frac{f(t+\epsilon t^{1-\alpha})-f(t)}{\epsilon},\quad \alpha\in(0,1],\quad t>0. \qquad (2)$$

This new definition satisfies the following properties.

Definition 2.2 Suppose that $c \geq 0$ and $t \geq c$, let h be a function defined on $(c,\ t]$ as well as $\alpha \in R$. Then, the $\alpha-$fractional integral of h is given by

$$ {}_tI_c^{\alpha} h(t) = \int_c^{\alpha} \frac{h(x)}{x^{1-\alpha}} dx, $$

if the Riemann improper integral exists.

Theorem 2.1. Let $\alpha \in (0,1]$, f, g be α-differentiable at a point t, then

- $D_t^{\alpha}(af + bg) = aD_t^{\alpha}(f) + bD_t^{\alpha}(g)$, for $a, b \in \mathbb{R}$.
- $D_t^{\alpha}(t^{\mu}) = \mu t^{\mu-\alpha}$, for $\mu \in \mathbb{R}$.
- $D_t^{\alpha}(fg) = fD_t^{\alpha}(g) + gD_t^{\alpha}(f)$.
- $D_t^{\alpha}(\frac{g}{g}) = \dfrac{gD_t^{\alpha}(f) - fD_t^{\alpha}(g)}{g^2}$.

Theorem 2.2. Let h be a differentiable function and _ is order of the conformable derivative. Let g be a differentiable function defined in the range of h, then

$$ D_t^{\alpha}(fog)(t) = t^{1-\alpha} g(t)^{\alpha-1} g'(t) D_t^{\alpha}\left(f(t)\right)_{t=g(t)}, $$

where "prime" is the classical derivatives with respect to t.

3. *Solution procedure*

To determine the solitary solution of equation (1), we first define the following new variables

$$ \begin{aligned} &u(x,y,t) = \hbar(\xi)e^{i\theta}, \xi = \left(\frac{1}{\alpha}\right)x^{\alpha} + \left(\frac{\mu}{\alpha}\right)y^{\alpha} - \left(\frac{\sigma}{\alpha}\right)t^{\alpha} \quad and \\ &\theta = \left(\frac{a}{\alpha}\right)x^{\alpha} + \left(\frac{b}{\alpha}\right)y^{\alpha} + \left(\frac{d}{\alpha}\right)t^{\alpha} + \theta_0, \end{aligned} \tag{3} $$

Substituting Eq. (3) in Eq. (1) and comparing real and imaginary parts, respectively, one can obtain

$$ \begin{gathered} \left(a^2 + 2b - d^2\right)\hbar - 2\hbar^3 + \left(\sigma^2 - 1\right)\hbar'' = 0, \\ \mu = -\left(a + d\sigma\right). \end{gathered} \tag{4} $$

Taking balance principles between $\hbar''$ and $\hbar^3$ into account in Eq. (4) yields $m = 1$. Immediately the general structure for the solution to the problem, which is presented in (4), is determined as follows

$$ \hbar(\xi) = B_1 \sin(\xi) + A_1 \cos(\xi) + A_0 \tag{5} $$

Now from method we have

$$\begin{aligned} &A_0 = 0, \\ &A_1 = \frac{\sqrt{3}}{6}\sqrt{6a^2 - 6d^2 - 6\sigma^2 + 12b + 6}, \\ &B_1 = \frac{1}{6}\sqrt{6a^2 - 6d^2 - 6\sigma^2 + 12b + 6}. \end{aligned} \tag{6}$$

Accurding to the steps of method we obtain the solutions of equation (1) as follows

$$\begin{aligned} u_1(x,y,t) =& \left[\frac{1}{6}\sqrt{6a^2 - 6d^2 - 6\sigma^2 + 12b + 6}\,\operatorname{sech}\left(\left(\frac{1}{\alpha}\right)x^\alpha + \left(\frac{\mu}{\alpha}\right)y^\alpha - \left(\frac{\sigma}{\alpha}\right)t^\alpha\right) + \right. \\ & \left.\frac{\sqrt{3}}{6}\sqrt{6a^2 - 6d^2 - 6\sigma^2 + 12b + 6}\,\tanh\left(\left(\frac{1}{\alpha}\right)x^\alpha + \left(\frac{\mu}{\alpha}\right)y^\alpha - \left(\frac{\sigma}{\alpha}\right)t^\alpha\right)\right] \times \\ & \exp\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha + \left(\frac{b}{\alpha}\right)y^\alpha + \left(\frac{d}{\alpha}\right)t^\alpha + \theta_0\right)\right), \end{aligned} \tag{7}$$

And dark singular soliton is

$$\begin{aligned} u_2(x,y,t) =& \left[\frac{1}{6}\sqrt{6a^2 - 6d^2 - 6\sigma^2 + 12b + 6}\,\operatorname{csch}\left(\left(\frac{1}{\alpha}\right)x^\alpha + \left(\frac{\mu}{\alpha}\right)y^\alpha - \left(\frac{\sigma}{\alpha}\right)t^\alpha\right) + \right. \\ & \left.\frac{\sqrt{3}}{6}\sqrt{6a^2 - 6d^2 - 6\sigma^2 + 12b + 6}\,\coth\left(\left(\frac{1}{\alpha}\right)x^\alpha + \left(\frac{\mu}{\alpha}\right)y^\alpha - \left(\frac{\sigma}{\alpha}\right)t^\alpha\right)\right] \times \\ & \exp\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha + \left(\frac{b}{\alpha}\right)y^\alpha + \left(\frac{d}{\alpha}\right)t^\alpha + \theta_0\right)\right), \end{aligned} \tag{8}$$

and

$$\begin{aligned} &A_0 = \frac{\sqrt{2a^2 - 2d^2 + 4b}}{2}, \\ &A_1 = \frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4}, \\ &B_1 = 0. \end{aligned} \tag{9}$$

optikal dark soliton solution is

$$\begin{aligned} u_3(x,y,t) =& \left[\frac{\sqrt{2a^2 - 2d^2 + \sigma^2 + 4b - 1}}{4}\tanh\left(\left(\frac{1}{\alpha}\right)x^\alpha + \left(\frac{\mu}{\alpha}\right)y^\alpha - \left(\frac{\sigma}{\alpha}\right)t^\alpha\right) + \right. \\ & \left.\frac{\sqrt{2a^2 - 2d^2 + 4b}}{2}\right]\exp\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha + \left(\frac{b}{\alpha}\right)y^\alpha + \left(\frac{d}{\alpha}\right)t^\alpha + \theta_0\right)\right), \end{aligned} \tag{10}$$

And dark singular soliton is

$$u_4(x,y,t) = \left[\frac{\sqrt{2a^2-2d^2+\sigma^2+4b-1}}{4}\coth\left(\left(\frac{1}{\alpha}\right)x^\alpha+\left(\frac{\mu}{\alpha}\right)y^\alpha-\left(\frac{\sigma}{\alpha}\right)t^\alpha\right)+\frac{\sqrt{2a^2-2d^2+4b}}{2}\right]\exp\left(i\left(\left(\frac{a}{\alpha}\right)x^\alpha+\left(\frac{b}{\alpha}\right)y^\alpha+\left(\frac{d}{\alpha}\right)t^\alpha+\theta_0\right)\right), \tag{11}$$

4. *Concluding remarks*

One of the main advantages of this method is the determination of different categories of solutions for the equation in a single framework; This means that the method can determine different types of solutions for the equation in a single process. Furthermore, one can easily deduce that the methods used in this study are very simple but very efficient methodologies for solving NPDEs.

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Evaluation of wavelet resolution merges for fusion of ASTER and RADARSAT SAR data

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ABSTRACT

In this study, the efficiency of different transforming techniques of wavelet resolution merge was evaluated for fusion of ASTER-L1B and RADARSAT-1 SAR data. In doing so, the SB (single band), IHS (intensity, hue, saturation) and PC (principal component) techniques were implemented. Since the suitability of the fusion algorithms for various applications depends on the quality of the fused images, several indicators were calculated to evaluate the fused images. The results based on standard deviation difference and root mean square error indicated advantage of the single band transforming technique, while the results of relative mean difference showed superiority of the PC technique. The results also indicated that the IHS is the worst technique based on our analyses. According to the achieved results, it can be concluded that all the transforming techniques preserved spectral quality of the original multispectral image, but the output of SB technique indicated slightly advantage than the outputs of PC and IHS in fusion of ASTER and RADARSAT SAR data. Finally, the results indicated capability of the wavelet resolution merge for multi-sensor data fusion.

Keywords
ASTER, RADARSAT SAR, wavelet resolution merge

1. INTRODUCTION

The concept of data fusion goes back to 1950's and 1960's, with searching for the realistic techniques of merging images from various sensors to generate a combined image which can be used to better identification of man-made and natural features (Wang et al., 2005). Image fusion can define as "the process of combining information from two or more images of a scene into a single composite image". The resultant image is more informative and is more suitable for visual perception or computer processing (Goshtasby and Nikolov, 2007). Image fusion techniques are commonly used to improve interpretability of the images, increase spatial resolution, enhance the certain features that are not visible in either of the single data alone, improve classification accuracy by complement data sets and substitute missing information (Pohl and Van Genderen, 1998).

There are three levels to perform image fusion: (i) pixel-based; (ii) object/feature-based; and (iii) decision-based. In pixel level fusion, a new image is formed whose pixel values can acquire by combining the pixel values of multi-sensor data. The resultant image is then used for further processing. In feature level fusion, the features are extracted from different types of images of the same geographic area. The extracted features are then classified using statistical or other types of classifiers. In decision level fusion, the images are processed separately. The processed information is then refined by combining the information obtained from different sources. The differences in information are resolved based on certain decision rules (Pohl and Van Genderen, 1998).

Pixel level image fusion was performed in this study. The pixel level fusion technique assumes correspondence between pixels in the input images; therefore, it is critical to accurately co-register all input images (Pohl and Van Genderen, 1998; Goshtasby and Nikolov, 2007). In this study, different transforming techniques of wavelet resolution merge were applied to fuse the ASTER-L1B and RADARSAT-1 SAR data. Since effectiveness of image fusion depends on the quality of the fused images, several indicators including the standard deviation difference (SDD), relative mean difference (RMD), and root mean square error (RMSE) were used to evaluate the performance of the fused images.

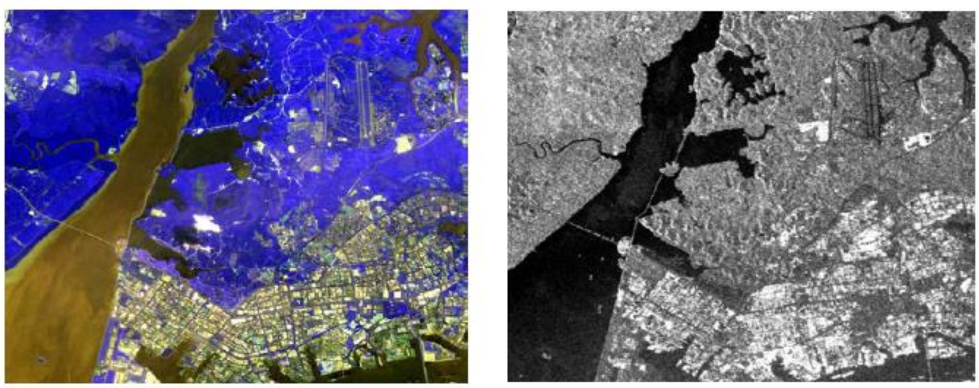
2. METHODS

2.1 Dataset

The RADARSAT-1 SAR wide mode data with the pixel size of 12.5m acquired on 23/10/2007 and the VNIR Bands (1, 2 and 3N) of ASTER-L1B (Registered Radiance at the Sensor V003) data with 15m pixel size acquired on 04/09/2002 were used as input data in this research.

2.2 Pre-processing

Data pre-processing is divided into three parts: (i) speckle noise reduction; (ii) co-registration; and (iii) resampling. In order to reduce speckle effect and improve interpretation capabilities of radar imagery, Lee algorithm with kernel windows size of 7×7 pixels and lines was applied to RADARSAT-1 SAR data. The Lee filter is a standard deviation based (sigma) filter that filters data based on statistics determined within individual filter windows. Lee filter preserves pixels boundaries while suppressing noise. The pixel being filtered is replaced by a value calculated using the surrounding pixels (Marghany and Hashim, 2010).
Accurate and precise co-registration of satellite images is the most important prerequisite for successful image fusion. To avoid the combination of unrelated data and achieve good result of fusion, the images should be precisely co-registered at the sub-pixel level. In this research, ASTER- L1B image is registered data, so used as the master for registration of RADARSAT-1 SAR data. By using image to image registration method, noise reduced RADARSAT SAR data with the root mean square error of less than one pixel was registered to the ASTER image while resampled to the pixel size of this image. Then, the datasets were resized to the same size using nearest neighbor method (Figure 1).



ASTER-L1B **RADARSAT-1 SAR**

Figure 1: ASTER-L1B and RADARSAT-1 SAR data used in this study

2.3 Wavelet resolution merge

The basic theory of wavelet transform is that an image could be divided into various high and low frequency components using various high and low pass filters. For instance, a low pass filter can use to create a low frequency image. Dividing this low frequency image from the original image could produce the corresponding high frequency image. These two created images are containing all information of original image and if add them together the result will be the original image. The same procedure could be done by high pass filter to derive low frequency image and adding the two images together would result the original image.
The wavelet family could be thought of as a high pass filter. Thus, wavelet-based high and low frequency images can be generated from any input image. By definition, the low frequency image is of lower resolution and the high frequency image is consists of the image details. This process could be repeated recursively. The generated low frequency image could be processed again with the kernels to produce new images with even lower resolution. Thus, starting with a 5-meter image would create a 10-meter low pass image and the corresponding high pass image. A 20-meter low and, corresponding, high pass images could be created in second iteration. Third recursion could create a 40-meter low and, corresponding, high pass images, etc. (Helmy et al., 2010).
Three transforming techniques were used. The simplest one is when the input low resolution image is only one band; for example, a single band of a multispectral image. This technique is known as SB (single band). The IHS (intensity,

hue, saturation) technique accepts only three input bands. It has been suggested for visual interpretation as the output image of this technique is the best for this purpose. Thus, the output of this technique could be appropriate for map production. The 3-band limitation of this technique can be resolved by sharpening the bands as separate groups of 3 and then layer stack back together the whole dataset. The PC (principal component) technique accepts any number of input image layers. The output of this technique better preserves the spectral fidelity of the input datasets. Therefore, it could be most appropriate for further processing of the data (Lemeshewsky, 2002).

2.4 Quality assessment methods

In this study, the transforming techniques of wavelet resolution merge were compared using different indicators including standard deviation difference (SDD), relative mean difference (RMD), and root mean square error (RMSE). The difference between standard deviation of the fused and original multispectral images is calculated. This difference indicates amount of variation due to fusion processing. The fused image with smallest possible shift in the standard deviation, best preserves the spectral information of the original image and indicates less distortion. The RMD is the calculation of relative difference of mean between the fused and original multispectral images. The fused image with the smallest possible relative mean difference best preserves the spectral information of the original multispectral image. The RMSE determines the spectral fidelity between the original image and the fused products. The RMSE measures the amount of changes per pixel due to the processing. Lower RMSE indicates lower distortion and better result of fusion.

3. RESULTS AND DISCUSSION

The outputs of wavelet resolution merge for ASTER and RADARSAT SAR data fusion are shown in Figure 2. Figure 2a is the output of principal component transforming technique, Figure 2b is the output of IHS and Figure 2c is the fused image of single band technique.

(a) Fused image of PC **(b) Fused image of IHS** **(c) Fused image of SB**

Figure 2: Fusion of ASTER and RADARSAT SAR data using wavelet resolution merge

In order to assess the quality of the fused images, several quantitative analysis criteria were calculated. These indicators were defined by comparing the fused images with the original multispectral image. The results are presented in Table 1. The ideal value for all proposed indicators is zero.

Table 1: Quantitative analysis criteria

	PC	IHS	SB	Original MS image
SDD	16.70	16.65	16.73	16.73
RMD	52.51	52.43	52.40	52.65
RMSE	0.069	0.130	0.067	

The results indicated that the bios of standard deviation differences between the PC, IHS and SB techniques with the original MS image are 0.03, 0.08 and 0, while the bios of relative mean differences are -0.0027, -0.0042 and -0.0047, respectively. It means, based on SDD, the SB technique performed better, but based on RMD, the PC technique was superior. In addition, the results showed that the RMSEs of SB and PC techniques are about 0.069 and 0.067 that are much better than the IHS technique with a RMSE of 0.130. Overall, based on the indicators calculated in this study, the SB is the preferred technique, PC is reliable, but IHS is the lowest accuracy technique. Based on the results of each indicator, the transforming techniques are sorted from the best to the worst as follows**:**

Based on the standard deviation difference: SB> PC >IHS
Based on the relative mean difference: PC>IHS>SB
Based on the root mean square error: SB>PC>IHS

4. CONCLUSION

In this study, different transforming techniques of wavelet resolution merge were applied to merge ASTER-L1B and RADARSAT-1 SAR satellite data. Since the effectiveness of the fusion techniques for various applications depends on the quality of the fused images, several indicators were used to examine performance of the fused images. Achieved results from the analyses of standard deviation difference and root mean square error indicated advantage of the single band technique, while the results of relative mean difference showed superiority of the PC technique. The results also indicated that IHS was the worst technique based on our analyses. Finally, based on our statistical analyses, it can be concluded that all the transforming techniques could preserve spectral quality of the original multispectral image, but the output of SB technique indicated slightly advantage than the outputs of PC and IHS for ASTER-L1B and RADARSAT-1 SAR data fusion.

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T-148

Identifying citrus and rice fields using NDVI and EVI indices from satellite images

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ABSTRACT

In this study, the suitability of Normalized Difference Vegetation Index (NDVI), Enhanced Vegetation Index (EVI), and Enhanced Vegetation Index 2 (EVI2) methods was evaluated for citrus and paddy race mapping from ASTER and Landsat satellite data. The results indicated that the accuracies of about 76% and 66% were obtained for the ASTER NDVI and ASTER EVI2, while the accuracies of Landsat NDVI and Landsat EVI for rice mapping was about 61% and 65%, respectively. The results revealed higher performance of ASTER NDVI for citrus mapping and Landsat EVI for rice mapping. The study concluded that obtaining a high accuracy result for rice mapping from medium resolution satellite images is difficult. However, for citrus mapping achieving a better result could be expected.

Keywords
NDVI, EVI, citrus, rice field

1. INTRODUCTION

Since a large amount of data about the nature of the Earth's surface have been collected by remote sensing satellites at different spatial, spectral, and temporal resolutions using appropriate band combinations, these data have become the main/primary sources extensively used for the extraction of vegetation and its changes over the Earth's surface in recent decades (Rokni and Musa, 2019). For vegetation detection and mapping from satellite images, usually satellite-derived vegetation indices are performed.

Vegetation index is defined as the arithmetic combination of two or more bands related to the spectral characteristics of vegetation (Matsushita et al., 2007). Several vegetation indices have been proposed for the enhancement and extraction of vegetation from satellite images, such as enhanced vegetation index (EVI) (Liu and Huete, 1995), normalized difference vegetation index (NDVI) (Rouse et al., 1973), soil-adjusted vegetation index (Huete, 1988), modified soil-adjusted vegetation index (Qi et al., 1994), weighted difference vegetation index (Clevers and Verhoef, 1993), etc. Among the existing vegetation indices, EVI and NDVI are the most popular indices used for the extraction of vegetation from satellite images. In this study, the NDVI and EVI indices are evaluated in detection and mapping citrus and paddy rice fields from ASTER L1B and Landsat-5 TM data. Accuracy assessment analysis was carried out by computing overall accuracy.

2. METHODS

2.1 Study area and dataset

The study area is Simorgh city, located in north of Iran between 30º 37′ N and 52º 53′ E of the Greenwich meridian (Figure 1). This city is covered by 10.5 thousand hectares of agricultural lands. More than 90% of this agricultural land is allocated to rice and citrus production. Irrigated rice and citrus production is common in this region and considered as a key source of income for around 100% of inhabitants (Golafshani et al., 2012). In addition, ASTER Level-1B data acquired on 27 May 2010 and Landsat -5 TM data acquired on 22 July 2010 were obtained from the US-Geological Survey (USGS) Global Visualization Viewer and were utilized for mapping citrus trees and paddy fields in this study.

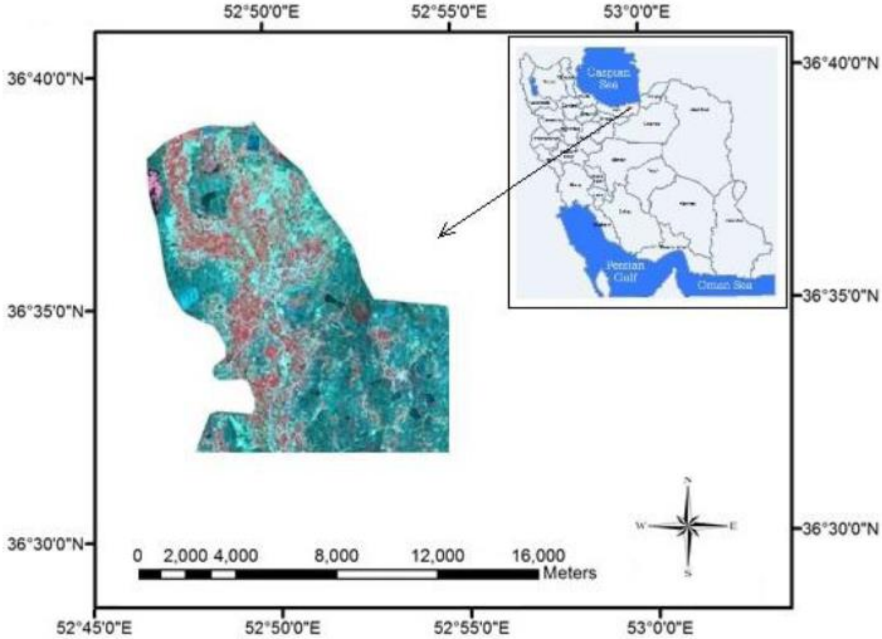

Figure 1: Location of the study area (resized ASTER image: 27 May 2010)

2.2 Satellite-derived indices

The NDVI and EVI indices were calculated from the ASTER and Landsat images to evaluate their performance in detection of citrus and rice fields. NDVI is calculated from the ASTER and Landsat images using the following formulas:

$$\text{NDVI ASTER} = \frac{\text{band 3N} - \text{band 2}}{\text{band 3N} + \text{band 2}} \qquad (1)$$

$$\text{NDVI Landsat} = \frac{\text{band 4} - \text{band 3}}{\text{band 4} + \text{band 3}} \qquad (2)$$

EVI was used to identify rice paddies because of its higher sensitivity to canopy structural variation. It is calculated as the formula (3). For sensors with no blue band, such as ASTER, a 2-band variation of EVI (EVI2) was developed (Jiang et al., 2008). EVI2 has been reported to correspond well with the original EVI. It was calculated for ASTER according to the formula (4):

$$\text{EVI} = 2.5 * [\frac{(\text{NIR} - \text{VISR})}{(\text{NIR} + 6 * \text{VISR} - 7.5 * \text{VISB} + 1)}] \qquad (3)$$

$$\text{EVI2} = 2.5 * [\frac{(\text{ASTER3N} - \text{ASTER2})}{(\text{ASTER3N} + 2.4 * \text{ASTER2} + 1)}] \qquad (4)$$

The accuracy of the generated maps was assessed quantitatively based on some ground truth points collected through field observation using Global Positioning System (GPS) instrument and calculating the overall accuracy.

3. RESULTS AND DISCUSSION

The density slicing followed by the trial and error was implemented to detect and map paddy rice and citrus fields in the study area. Because the acquisition time of the ASTER image was not suitable for rice mapping, thus this image was used for citrus mapping, and Landsat image was used for rice mapping. The citrus and paddy rice maps generated from ASTER and Landsat images are shown in Figure 2.

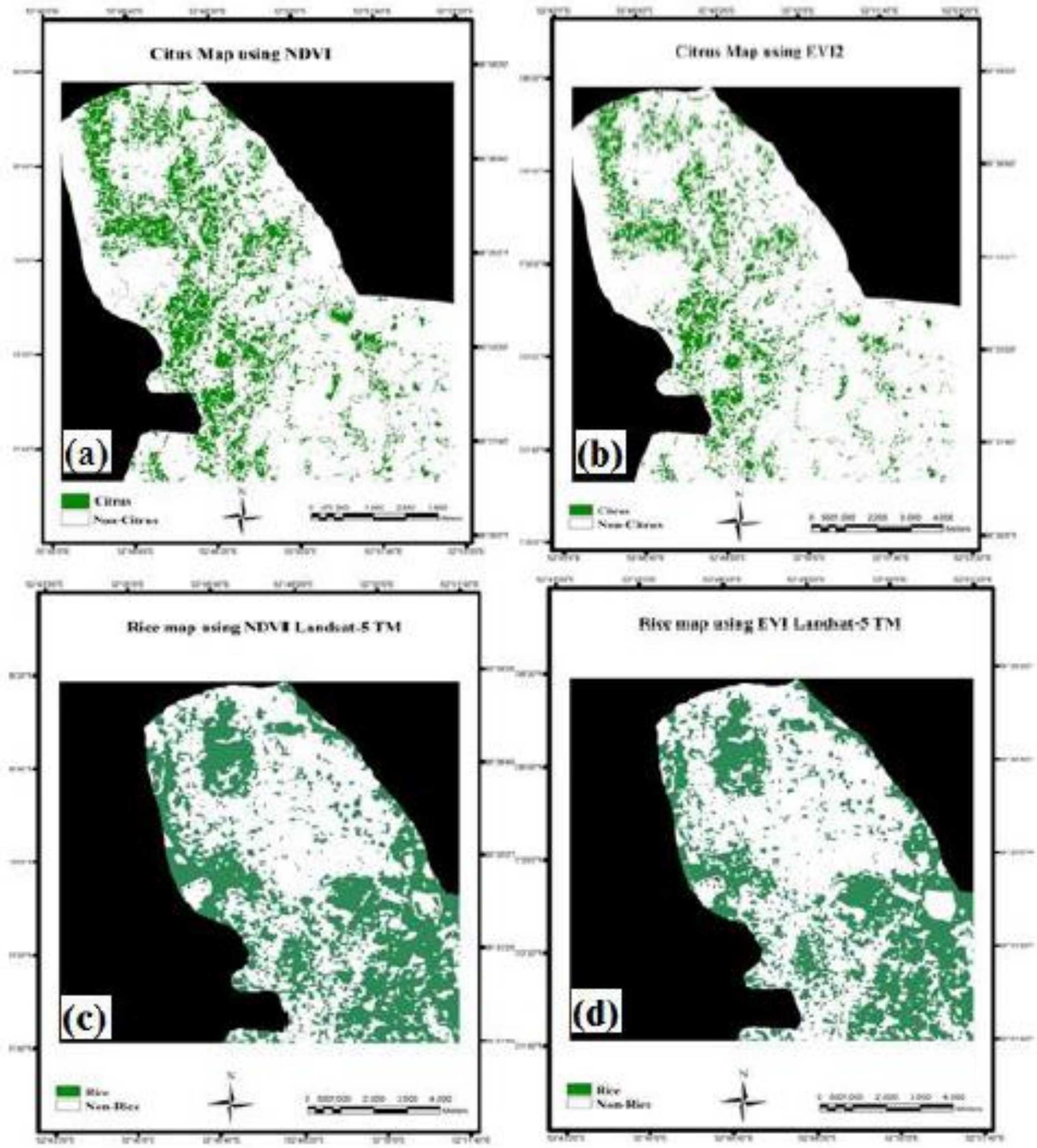

Figure 2: Citrus and rice maps generated using: (a) ASTER NDVI, (b) ASTER EVI2, (c) Landsat NDVI, and (d) Landsat EVI

In order to assess the accuracy of the generated maps, overall accuracy for each map was calculated, as presented in Table 1. The results indicated that the overall accuracies of citrus maps generated from ASTER NDVI and ASTER EVI2 are about 76% and 66%, respectively. It reveals higher performance of ASTER NDVI for citrus mapping. Moreover, the results indicated that the accuracies of Landsat NDVI and Landsat EVI for rice mapping are about 61% and 65%, that demonstrated higher performance of Landsat EVI for this purpose. The results proved suitability of NDVI for citrus mapping; inverse, higher performance of EVI for rice mapping. The study concluded that even by acquiring the satellite image in suitable time, obtaining a high accuracy rice map is difficult. However, for citrus mapping, the accuracy could be higher.

Table 1: Overall accuracy of the generated maps

Method	Overall accuracy (%)
Citrus map from ASTER NDVI	76
Citrus map from ASTER EVI2	66
Rice map from Landsat NDVI	61
Rice map from Landsat EVI	65

4. CONCLUSION

In this study, the performance of ASTER NDVI and EVI2 was evaluated for citrus mapping, while the suitability of Landsat NDVI and EVI was assessed for rice mapping in Simorgh, Iran. The accuracy of the generated maps was evaluated by calculating overall accuracy. The results indicated higher performance of ASTER NDVI than ASTER EVI2 for citrus mapping, while the accuracy of Landsat EVI was higher than Landsat NDVI for rice mapping. The study concluded that even by acquiring the satellite image in suitable time, it is difficult to identify rice fields with high accuracy. However, the accuracy could be higher for citrus mapping.

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